پرهام سراجی

1) $g(x) = f(\tan^2 x + \sqrt{2}\cos x)$, $g'(\frac{\pi}{4}) = \sqrt{3}$

$g'(x) = f'(\tan^2 x + \sqrt{2}\cos x) \times (2\tan x(1+\tan^2 x) - \sqrt{2}\sin x)$

$\underbrace{g'(\frac{\pi}{4})}_{\sqrt{3}} = f'(1+1) \times (2(2)-1)$

$f'(2) = \frac{\sqrt{3}}{3}$

2) $f'(x) = \frac{8+\cos^3 x}{4-\cos^2 x}$, $g(x) = \frac{2}{2-\cos x}$

$\underbrace{f'(\frac{7\pi}{6}) - 2g'(\frac{7\pi}{6})}_{(f-2g)'(\frac{7\pi}{6})}$

$f(x) = \frac{(2+\cos x)(4+\cos^2 x - 2\cos x)}{(2-\cos x)(2+\cos x)}$

$= \frac{4+\cos^2 x - 2\cos x}{2-\cos x}$

$(f-2g)(x) = \frac{\cos^2 x - 2\cos x}{2-\cos x} = \frac{\cos x(\cos x - 2)}{2 - \cos x}$

$= -\cos x$

$(f-2g)'(x) = \sin x$, $\sin(\frac{7\pi}{6}) = -\frac{1}{2}$

3) $f(x) = |4x-3|\sqrt{ax}$, $f'_+(\frac{3}{4}) - f'_-(\frac{3}{4}) = 2\sqrt{6}$

$f(\frac{3}{4}^+) = (4x-3)\sqrt{ax} \Rightarrow f'(\frac{3}{4}) = (4)(\sqrt{\frac{3}{4}a}) + \frac{a}{2\sqrt{\frac{3}{4}a}}(4a-a)$

$f(\frac{3}{4}^-) = (3-4x)\sqrt{ax} \Rightarrow f'_-(\frac{3}{4}) = -4(\sqrt{\frac{3}{4}a}) + \frac{a}{2\sqrt{\frac{3}{4}a}}(3-4a)$

$\Rightarrow 8(\sqrt{\frac{3}{4}a}) = \frac{8\sqrt{3a}}{2} = 2\sqrt{6} \Rightarrow 4\sqrt{3a} = 2\sqrt{6}$

$12a = 6 \Rightarrow a = \frac{1}{2}$

/ /

۴) $f(۴) + f'(۴) = -1$ $\quad f'(۴) = -a$, $f(۴) = ۲-۴a$ $\Big\}$ $۲-۵a \Rightarrow ۵a = ۳ \Rightarrow a = ۰,۶$

$f'(۴) = -۰,۶$

۵) $y = x + ۵ \Rightarrow ۶x = ۵ \Rightarrow \frac{۵}{۶} = x$ $\quad y' = \frac{1}{۷} = \frac{a+۳}{(۳x+۲)}$

$\Rightarrow \frac{۲}{۵} \times \frac{۷}{۲} \times \frac{1}{۷} = a + ۳ \Rightarrow a = \frac{۷}{۳} - ۳ = -\frac{۵}{۳}$

۶) ~~[illegible]~~ $\quad f(۲) = ۰ \quad ۸ + ۲a + b = ۰$

$f'(۲) = ۰ \quad ۳a^۲ + a = ۰ \Rightarrow a = -۴$

$b - a = ۲۸ \qquad b = ۱۶$

۷) $f(x) = \sin^۲(x^۲) \Rightarrow f'(x) = ۲xn \sin^{n-1}(x^۲)\cos(x^۲)$

$\Rightarrow \lim \frac{f(x) f'(x)}{(1-\cos x)^{m}} = \lim_{x \to 0} \frac{\sin^۲(x^۲)\, ۲xn \cos(x^۲)\sin^{n-1}(x^۲)}{(1-\cos x)^{m}}$

$= \lim \frac{۲nx \sin^{۲x-1}(x^۲)}{(\frac{x^۲}{۲})^{m}}$ $\quad \lim \frac{۲n x^{۴x-1}}{\frac{x^۲}{۲m}} = \lim_{x \to 0} \frac{۲^{x+1} n x^{۴n-1}}{۲m}$

$n = ۲ \quad m = ۳k$

$= ۳۲\sqrt{۲} \Rightarrow ۲m = ۴n - 1 \qquad ۲m + n = ۹$

۸) $f(x) = \frac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow y\sqrt{x} - y = \sqrt{x} + 1 \Rightarrow \sqrt{x}(y-1) = y+1$

$\Rightarrow \sqrt{x} = \frac{y+1}{y-1} \Rightarrow x = \left(\frac{y+1}{y-1}\right)^۲ \Rightarrow f^{-1}(x) = ۲\left(\frac{x+1}{x-1}\right)\left(\frac{-1-1}{(x-1)^۲}\right)$

$\Rightarrow x = ۲ \Rightarrow ۶(-۲) = -۱۲$

۹) $f(x) = (x^۲+1)^۳(ax+1)$ $\quad a = -1 \quad f(x) = ۸(-a+1)$; $\quad a = ۰ \quad f(x) = 1$

$\frac{1-(-۸a+۸)}{a+1} = 1 + ۸a - ۸ = ۸a - ۷ = ۱۱ \Rightarrow a = -\frac{1}{۲}$

$f'(1) = ۳(x^۲+1)^۲(۲x)(-\frac{1}{۲}x+1) + (-\frac{1}{۲})(x^۲+1)^۳ = ۳(۴)(۲)(\frac{1}{۲}) - ۴ = ۸$

/ /

$y = \sin x + \cos^2 x$ — $x = \frac{\pi}{4}$: $y = \frac{\sqrt{2}}{2} \times 0 = 0$; $y = 1\pi - 1 = -1$

$\frac{-1 - 0}{\frac{\pi}{2} - \frac{\pi}{4}} = \frac{-1}{\frac{\pi}{4}} = \frac{-4}{\pi}$ (1c

$y = \sin^2 x - \cos x$ — $x = \frac{\pi}{2}$; $\frac{\pi}{4}$; $\frac{\pi}{2}$; 1

$\frac{1}{2} - \frac{1}{2} = 0$

$\frac{1}{\frac{\pi}{4}} = \frac{4}{\pi}$

$-\frac{4}{\pi} \times \frac{\pi}{4} = -1$