پرهام پیراوی

۱۹ آفرین

۱)

$g(x) = f(\tan^2 x + \sqrt{3}\cos x)$, $g'(\frac{\pi}{6}) = \sqrt{3}$

$g'(x) = f'(\tan^2 x + \sqrt{3}\cos x) \times (2\tan x(1+\tan^2 x) - \sqrt{3}\sin x)$

$g'(\frac{\pi}{6}) = f'(1+1) \times (2(2) - 1)$

$\sqrt{3} \qquad f'(2) = \frac{\sqrt{3}}{3}$ ✓ ۲

۲)

$f(x) = \frac{8+\cos^3 x}{4-\cos^2 x}$, $g(x) = \frac{3}{2-\cos x}$

$f'(\frac{7\pi}{6}) - 2g'(\frac{7\pi}{6})$

$(f-2g)'(\frac{7\pi}{6})$

$f(x) = \frac{(2+\cos x)(4+\cos^2 x - 2\cos x)}{(2-\cos x)(2+\cos x)}$

$= \frac{4+\cos^2 x - 2\cos x}{2-\cos x}$

$(f-2g)(x) = \frac{\cos^2 x - 2\cos x}{2-\cos x} = \frac{\cos x(\cos x - 2)}{2 - \cos x}$ ۲

$= -\cos x$

$(f-2g)'(x) = \sin x \qquad \sin(\frac{7\pi}{6}) = -\frac{1}{2}$ ✓

۳)

$f(x) = |4x-3|\sqrt{ax}$, $f'_+(\frac{3}{4}) - f'_-(\frac{3}{4}) = 2\sqrt{6}$

$f(\frac{3}{4}^+) = (4x-3)\sqrt{ax} \Rightarrow f'_+(\frac{3}{4}) = (4)(\sqrt{\frac{3}{4}a}) + \frac{a}{2\sqrt{\frac{3}{4}a}}(4a-a)$

$f(\frac{3}{4}^-) = (3-4a)\sqrt{ax} \Rightarrow f'_-(\frac{3}{4}) = -4(\sqrt{\frac{3}{4}a}) + \frac{a}{2\sqrt{\frac{3}{4}a}}(3-4a)$

$\Rightarrow 8(\sqrt{\frac{3}{4}a}) = \frac{8\sqrt{3a}}{2} = 2\sqrt{6} \Rightarrow 4\sqrt{3a} = 2\sqrt{6}$ ۲

$12a = 6 \Rightarrow a = \frac{1}{2}$

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$f(۴)+f'(۴)=-۱$ $\quad f'(۴)=-a,\ f(۴)=۲-۴a \Big\} ۲-۵a \Rightarrow ۵a=۳ \Rightarrow a=۰,۶$ (۴

$f'(۴)=-۰,۶$ ✓ (۲)

$x=y$

$۷\ y=x+۵ \Rightarrow ۶x=۵ \Rightarrow \frac{۵}{۶}=x \qquad y'=\frac{۱}{۷}=\frac{a+۳}{۴x+۲}$ (۵

$\Rightarrow \frac{۲}{۵}\times\frac{۷}{۲}\times\frac{۱}{۷}=a+۳ \Rightarrow a=\frac{۷}{۴}-۳=-\frac{۵}{۴}$ (۱,۲۵)

$۸-۲۴-b=۰ \rightarrow b=-۱۶$

$f(۲)=۰ \qquad ۸+۲a+b=۰$ (۶

دقت! (۱,۷۵)

$f'(۲)=۰ \qquad ۳a^۲+a=۰ \Rightarrow a=-۴$

$b-a=۲۸ \qquad b=~~۱۶~~-۱۶$

$-۱۶-(-۱۲)=-۴$

$f(x)=\sin^۲(x^۲) \Rightarrow f'(x)=۲xn\sin^{n-۱}(x^۲)\cos(x^۲)$ (۷

$\Rightarrow \lim \frac{f(x)f'(x)}{(۱-\cos x)^{m}}=\lim_{x\to ۰}\frac{\sin^۲(x^۲)\,۲xn\cos(x^۲)\sin^{n-۱}(x^۲)}{(۱-\cos x)^{m}}$

$=\lim \frac{۲nx\sin^{۲+n-۱}(x^۲)}{(\frac{x^۲}{۲})^{m}} \qquad \lim \frac{۲nx^{۴n-۱}}{\frac{x^۲}{۲^m}}=\lim_{x\to ۰}\frac{۲^{x+۱}nx^{۴n-۱}}{۲m}$ (۲)

$=۳۲\sqrt{۲} \Rightarrow ۲m=۴n-۱ \qquad ۲m+n=۹$ ✓ $\quad n=۲ \quad m=۳/۵$

$f(x)=\frac{\sqrt{x}+۱}{\sqrt{x}-۱} \Rightarrow y\sqrt{x}-y=\sqrt{x}+۱ \Rightarrow \sqrt{x}(y-۱)=y+۱$ (۸

$\Rightarrow \sqrt{x}=\frac{y+۱}{y-۱} \Rightarrow x=\left(\frac{y+۱}{y-۱}\right)^۲ \Rightarrow f^{-۱}(x)=۲\left(\frac{x+۱}{x-۱}\right)\left(\frac{-۱-۱}{(x-۱)^۲}\right)$

$\Rightarrow x=۲ \Rightarrow ۶(-۲)=-۱۲$ ✓ (۲)

$f(x)=(x^۲+۱)^۳(ax+۱)$ $\begin{cases} a=-۱ & f(x)=۸(-a+۱) \\ a=۰ & f(x)=۱ \end{cases}$ (۹

$\frac{۱-(-۸a+۸)}{a+۱}=۱+۸a-۸=۸a-۷=۱ \Rightarrow a=-\frac{۱}{۲}$ (۲)

$f'(۱)=۳(x^۲+۱)^۲(۲x)(-\frac{۱}{۲}x+۱)+(-\frac{۱}{۲})(x^۲+۱)^۳=۳(۴)(۲)(\frac{۱}{۲})-۴=۸$ ✓

/ /

$y = \sin x + \cos^2 x$

$x_0 = \frac{\pi}{4}$ $\quad y = \frac{\sqrt{2}}{2} + 0 = 0$

$x_0 = \frac{\pi}{2}$ $\quad y = 1 + 0 - 1 = -1$

$y = \sin^2 x \cdot \cos x$

$\frac{\pi}{2}$

$\frac{\pi}{4}$ $\quad 1$

$\frac{1}{\pi} \cdot \frac{1}{\pi} = 0$

$\frac{-1 - 0}{\frac{\pi}{2} - \frac{\pi}{4}} = \frac{-1}{\frac{\pi}{4}} = -\frac{4}{\pi}$ (1c

$\frac{1}{\frac{\pi}{2}} \cdot \frac{\pi}{\pi^2}$ $\quad -\frac{\pi}{\pi^2} \times \frac{\pi}{\pi} = -1$

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