پاسخ نامه تشریحی - تألیف سعید ۲۳۰۰   بنام خدا   دوازدهم پسر B

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$g(x)=f(\tan^2x+\sqrt2\cos x)\to g'(x)=2(1+\tan^2x)\tan x-\sqrt2\sin x$

$x=\frac{\pi}{4}\to g'(\frac{\pi}{4})=2(1+1).1-\frac{\sqrt2\sqrt2}{2}f'(1+\frac{\sqrt2\sqrt2}{2})\to 3f'(2)=\sqrt3\to f'(2)=\frac{\sqrt3}{3}$

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$g(x)=\frac{2}{2-\cos x}\qquad g'(x)=\frac{-2\sin x}{(2-\cos x)^2}\to g'(\frac{7\pi}{6})=\frac{4}{16+8\sqrt3}$

$f(x)=\frac{8+\cos^3x}{4-\cos^2x}\to f'(x)=\frac{(3x-\sin x\cos^2x)(4-\cos^2x)-(2\sin x\cos x)}{(4-\cos^2x)^2}$

$f'(\frac{7\pi}{6})=-\frac{1}{2}$

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$f'(x)\to\lim_{x\to\frac{3}{4}^+}\frac{(4x-3)(\sqrt{ax})}{x-\frac{3}{4}}=-4\sqrt{\frac{3a}{4}}=-2\sqrt{3a}\qquad 2\sqrt{3a}-(-2\sqrt{3a})=2\sqrt6$

$4\sqrt{3a}=2\sqrt6\to\sqrt{3a}=\sqrt{\frac{6}{4}}\Rightarrow a=\frac{1}{2}$

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$f'(4)=-a,\ f(4)=2-4a\qquad f(4)+f'(4)=-1\to 2-4a-a=-1\to a=\frac{3}{5}\to f'(4)=-0.6$

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$y=x-\frac{a}{7}=\frac{ax-1}{3x-1}\to 3x^2+(16-7a)x+12=0\xrightarrow{\text{ریشه مضاعف}}(16-7a)^2=144\to 16-7a=12$

$a=\frac{4}{7}$ ✗ $\to 16-7a=-12\to a=4$

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$f(2)=0\to 8+4a-b=0\to 8-24-b=0\to b=-16\to f'(2)=12+a=0\quad a=-12$

$b-a=-16-(-12)=-4$

پاسخ‌های تشریحی

۷- $f(x)=\sin^n(x^2)$ $\quad \lim \dfrac{f(x)f'(x)}{(1-\cos x)^m}=\text{عدد}$ $\quad \lim \dfrac{2nx\cos x^2 \sin^{2n-1}(x^2)}{(1-\cos x)^m}$

$f'(x)=n\sin^{n-1}(x^2)\cos(x^2)\times 2x \xrightarrow{\sin x\approx x} \,,\, 1-\cos x\approx \frac{x^2}{2}$ $\quad \dfrac{(2nx)(x^2)^{2n-1}}{\dfrac{x^{2m}}{2^{2m}}}$

$x^{2m} \to x^{4n-1} \to 2m=4n-1 \to 2^{m+2}$ عدد $=2 \to m=$ فرد

$2m+n=7+2=9 , n+2^{m+1}=2^1=2 \Rightarrow n=2$

۸- $y=\dfrac{\sqrt{x}+1}{\sqrt{x}-1} \to y\sqrt{x}-y=\sqrt{x}+1 \to \sqrt{x}(y-1)=y+1 \to \sqrt{x}=\dfrac{y+1}{y-1} \to x=\left(\dfrac{y+1}{y-1}\right)^2$

$f^{-1}(x)=\left(\dfrac{x+1}{x-1}\right)^2 \to f^{-1'}(x)=2\left(\dfrac{x+1}{x-1}\right)\left(\dfrac{x-1-(x+1)}{(x-1)^2}\right) \Rightarrow f^{-1'}(2)=-12$

۹- $\dfrac{f(0)-f(-1)}{0-(-1)}=\dfrac{1-(8(1-a))}{1}=8a-7 \to a=-\dfrac{1}{2}$ $\quad f(x)=(x^2+1)^4\left(-\dfrac{x}{2}+1\right)$

$f'(x)=4(2x)(x^2+1)^3\left(-\frac{x}{2}+1\right)-\frac{1}{2}(x^2+1)^4 \to f'(1)=8$

۱۰- $y=\sin x\cos 3x \to \left[\dfrac{\pi}{4},\dfrac{\pi}{2}\right] \to \dfrac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}=\dfrac{-1}{\frac{\pi}{4}}=-\dfrac{4}{\pi}$

$y=\sin^4 x-\cos^4 x \to \dfrac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}=\dfrac{4}{\pi}$ $\quad \dfrac{-\frac{4}{\pi}}{\frac{4}{\pi}}=1$