پرهام ستاری - تکلیف سری ۲۳،۱۰ — بنام خدا — دوازدهم پسر B

۲۰ آفرین

-۱
$g(x)=f(\sin^2 x+\sqrt{2}\cos x) \to g'(x)=2(1+\sin^2 x)\sin x-\sqrt{2}\sin x$

$x=\frac{\pi}{4} \to g'(\frac{\pi}{4})=2(1+1)\cdot 1-\frac{2\sqrt{2}}{2}f'(1+\frac{2\sqrt{2}}{2}) \to 3f'(2)=\sqrt{3} \to f'(2)=\frac{\sqrt{3}}{3}$

-۲
$g(x)=\frac{2}{2-\cos x}$ $\quad g'(x)=\frac{-2\sin x}{(2-\cos x)^2} \to g'(\frac{7\pi}{6})=\frac{4}{19+8\sqrt{3}}$

$f(x)=\frac{8+\cos^3 x}{4-\cos^2 x} \to f'(x)=\frac{(3x-\sin x\cos^2 x)(4-\cos^2 x)-(2\sin x\cos x)}{(4-\cos^2 x)}$

$f'(\frac{7\pi}{6})=-\frac{1}{2}$

-۳
$f'(x) \to \lim_{x\to\frac{3}{4}^+}\frac{(4x-3)(\sqrt{ax})}{x-\frac{3}{4}}=-4\sqrt{\frac{3a}{4}}=-2\sqrt{3a}$ $\quad 2\sqrt{3a}-(-2\sqrt{3a})=2\sqrt{6}$

$4\sqrt{3a}=2\sqrt{6} \to \sqrt{3a}=\sqrt{\frac{6}{4}} \Rightarrow a=\frac{1}{2}$

-۴
$f'(4)=-a,\ f(4)=2-4a$ $\quad f(4)+f'(4)=-1 \to 2-4a-a=-1 \to a=\frac{3}{5} \to f'(4)=-0.6$

-۵
$y=x-\frac{a}{7}=\frac{ax-1}{3x-1} \to 3x^2+(16-7a)x+12=0$ $\xrightarrow{\text{ریشه مضاعف}} (16-7a)^2=144 \to 16-7a=12$

$a=\frac{4}{7}$ ✗ $\to 16-7a=-12 \to a=4$

-۶
$f(2)=0 \to 8+2a-b=0 \to 8-24-b=0 \to b=-16 \to f'(2)=12+a=0,\ a=-12$

$b-a=-16-(-12)=-4$

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$f(x)=\sin^n(x^2)$ $\quad \lim \dfrac{f(x)f'(x)}{(1-\cos x)^m}=c\sqrt{2}$ $\quad \lim \dfrac{2nx\cos x^2\sin^{2n-1}(x^2)}{(1-\cos x)^m}$

$f'(x)=n\sin^{n-1}(x^2)\cos(x^2)\times 2x \xrightarrow{\sin x\approx 0} 1-\cos x\approx \dfrac{x^2}{2}$ $\quad \dfrac{(2nx)(x^2)^{2n-1}}{\dfrac{x^{2m}}{2^m}}$

$x^{2m}\to x^{4n-1}\to 2m=4n-1\to 2^{m+1}=2 \to m=1,5$ غلط

$2m+n=7+2=9,\ n=2^{m+1}=2\to 2\neq n=2$

(۲)

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$y=\dfrac{\sqrt{x}+1}{\sqrt{x}-1}\to y\sqrt{x}-y=\sqrt{x}+1\to\sqrt{x}(y-1)=y+1\to\sqrt{x}=\dfrac{y+1}{y-1}\to x=\left(\dfrac{y+1}{y-1}\right)^2$

$f^{-1}(x)=\left(\dfrac{x+1}{x-1}\right)^2\to f^{-1\,\prime}(x)=2\left(\dfrac{x+1}{x-1}\right)\left(\dfrac{x-1-(x+1)}{(x-1)^2}\right)\Rightarrow f^{-1\,\prime}(2)=-12$

(۲)

۹-

$\dfrac{f(0)-f(-1)}{0-(-1)}=\dfrac{1-(8(1-a))}{1}=8a-7\to a=-\dfrac{1}{2}$ $\quad f(x)=(x^2+1)^4\left(-\dfrac{x}{2}+1\right)$

$f'(x)=4(2x)(x^2+1)^3\left(-\dfrac{x}{2}+1\right)-\dfrac{1}{2}(x^2+1)^4\to f'(1)=8$

(۲)

۱۰-

$y=\sin x\cos x\to\left[\dfrac{\pi}{4},\dfrac{\pi}{2}\right]\to\dfrac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}=\dfrac{-\frac{1}{2}}{\frac{\pi}{4}}=-\dfrac{2}{\pi}$

(۲)

$y=\sin^4 x-\cos^4 x\to\dfrac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}=\dfrac{4}{\pi}$ $\quad \dfrac{-\frac{4}{\pi}}{\frac{4}{\pi}}=-1$

دقت کن

$\dfrac{-\frac{4}{\pi}}{\frac{4}{\pi}}=-1$