14,5

نام و نام خانوادگی ......علی درخشان‌فر........ پاسخنامه تشریحی تکلیف شماره .... کلاس ....B......

(1) $g'(\frac{\pi}{4}) = (2\times\tan\frac{\pi}{4}\times\frac{1}{\cos^2\frac{\pi}{4}} + (-\sqrt{2}\sin\frac{\pi}{4})\times f'(2) = \sqrt{3}$ ۱

$= 3f'(2) = \sqrt{3} \Rightarrow \boxed{f'(2) = \frac{\sqrt{3}}{3}}$

2

مشتق‌های استفاده شده:

$\cos'(u) = -u'\times \sin u$

$\tan'(u) = \sec^2(u)\times u'$

(1) $f(x) = \frac{(\cos x+2)(\cos^2 x+4-2\cos x)}{(2-\cos x)(2+\cos x)} = \cos x - 2 \xrightarrow{f'(x)} -\sin x$ ۲

(2) $g'(x) = -\frac{2\sin x}{(\cos x-2)^2}$

(3) $f'(\frac{7\pi}{6}) - 2g'(\frac{7\pi}{6}) = -\sin\frac{7\pi}{6} + \frac{4\sin\frac{7\pi}{6}}{(\cos\frac{7\pi}{6}-2)^2} = \boxed{\frac{21+8\sqrt{3}}{4}}$

1,25

(1) $f'_+(\frac{3}{4}) = (4x-3)\sqrt{ax} = 4\sqrt{ax}+0 = 4\sqrt{\frac{3}{4}a}$ ۳

(2) $f'_-(\frac{3}{4}) = (3-4x)\sqrt{ax} = -4\sqrt{ax}+0 = -4\sqrt{\frac{3}{4}a}$

(3) $8\sqrt{\frac{3}{4}a} = 2\sqrt{6} \quad \sqrt{6} = \sqrt{\frac{3}{4}a} \to 6 = \frac{3}{4}a \quad \boxed{a=8}$

1,25

(1) $f(4) = -4a+2 , f'(4) = -a \Rightarrow -4a+2-a = -1$ ۴

$-5a = -3 \quad a = \frac{3}{5}$

(2) $f'(4) = \boxed{-a = -\frac{3}{5}}$

2

~~$(1)\ f' = \frac{a(3x+1)-3(ax-1)}{(3x+1)^2} = \frac{3ax+a-3ax+3}{(3x+1)^2} = \frac{1}{7}$~~ ۵

~~$9x^2+6x\ldots = 7a+\ldots$~~

(1) $\frac{ax-1}{3x+1} = \frac{x+5}{7} \to 7ax-7 = 3x^2+16x+5$

(2) $3x^2+(16-7a)x+12=0$

2

چون نقاط تلاقی باید مضاعف باشد $\rightarrow$ کل معادله $= 3(x-2)^2 \Rightarrow -7a+16 = -12 \Rightarrow \boxed{a=4}$

$a=4$

علی درخشان فر

① مشتق در $x=2$ صفر است $\Rightarrow 3x^2+a=0 \to 3\times(4)+a=0 \to a=-12$

② $f(2)=0 \Rightarrow 8-12(2)-b=0 \quad b=-16$

$b-a=-16+12=\boxed{-4}$ ۲ ۶

$$\frac{\sqrt{x}+1}{\sqrt{x}-1}=2 \to 2\sqrt{x}-2=\sqrt{x}+1 \to \sqrt{x}=3 \Rightarrow x=9$$

$$f'(9)=\frac{\frac{1}{2\times 3}(3-1)-(4)\times\frac{1}{2\times 3}}{4}=\frac{\frac{1}{3}-\frac{2}{3}}{4}=-\frac{1}{12}$$ ۸

$\boxed{\text{جواب} = -12}$ ۲

$$\frac{f(a)-f(-1)}{a-(-1)}=8a-7=-11 \Rightarrow a=-\frac{1}{2}$$

$$f'(x)=3\times 2x\times(x^2+1)^2\times\left(-\frac{x}{2}+1\right)-\frac{1}{2}(x^2+1)^3 \Rightarrow f'(1)=\boxed{8}$$ ۲ ۹

② $y_2'=2\sin x$ ۱۰

① $\dfrac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}=\dfrac{-1}{\frac{\pi}{4}}=\dfrac{4}{\pi}$ ③ $\dfrac{-\frac{4}{\pi}}{\frac{4}{\pi}}=-1$ * ۲

② $\dfrac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}} \xrightarrow{y_2''=-\cos x} \dfrac{1}{\frac{\pi}{4}}=\dfrac{4}{\pi}$

۲

$$f-2g\left(\frac{7\pi}{4}\right)=\frac{(2+\cancel{\cos x})(\cos^2 x-2\cos x+4)}{(2-\cos x)(2+\cancel{\cos x})}-\frac{4}{2-\cos x}=\frac{\overset{\cos x(\cancel{\cos x}-2)}{\cos^2 x\cancel{-2\cos x}}}{2-\cancel{\cos x}}=-\cos x$$

$$(f-2g)'=-(\cos x)'=+\sin x \longrightarrow \sin\left(\frac{7\pi}{4}\right)=\boxed{-\frac{1}{2}}$$

۳

$$x\to \frac{3}{4}^{+} \rightsquigarrow f(x)=4x-3\sqrt{\frac{3a}{4}} \xrightarrow[\text{صفرشونده}]{\text{مشتق عبر}} f'(x)=\frac{4\sqrt{3a}}{2}=2\sqrt{3a}$$

$$x\to \frac{3}{4}^{-} \rightsquigarrow f(x)=-4x+3\sqrt{\frac{3a}{4}} \xrightarrow[\text{صفرشونده}]{\text{مشتق عبر}} f'(x)=\frac{-4\sqrt{3a}}{2}=-2\sqrt{3a}$$

$$4\sqrt{3a}=2\sqrt{6} \rightarrow \sqrt{3a}=\frac{\sqrt{6}}{2} \rightarrow \boxed{a=\frac{1}{2}}$$

۷

$$\lim_{x\to 0}\frac{\sin^n(x^2)\times 2x\sin^{n-1}(x^2)\times\cos x^2}{\left(2\sin^2\frac{x}{2}\right)^m} \xrightarrow{\text{هم‌ارزی}} \lim_{x\to 0}\frac{(x^2)^n\times n\times(x^2)^{n-1}\times 2x}{\left(2\left(\frac{x}{2}\right)^2\right)^m}=\frac{(x)^{2n+2n-2+1}\times 2n}{(x)^{2m}\times\frac{1}{2}}$$

$$=\frac{2nx^{4n-1}}{\frac{1}{2^m}\times x^{2m}}=2^{m+1}\times n\times x^{4n-2m-1}$$

$$\rightarrow 4n-2m-1=0 \longrightarrow n=\frac{2m+1}{4}$$

$$\rightarrow 2^{m+1}\times\frac{2m+1}{4}=32\sqrt{2} \longrightarrow m=\frac{7}{2},\ n=2 \rightarrow \boxed{2m+n=9}$$