نام و نام خانوادگی ..امیرعباس پوراسماعیل......... پاسخنامه تشریحی تکلیف شماره ...۲۳.... کلاس ..دوازدهم ریاضی B........

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$g(x) = f(\tan^2 x + \sqrt{2}\cos x)$

$g'(x) = f'(\tan^2 x + \sqrt{2}\cos x) \times \left(2 \times \left(\frac{\cos x}{-\sin x}\right)(\tan x) + \sqrt{2} \times (-\sin x)\right)$

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$f(x) = \frac{8+\cos^3}{4-\cos^2} = \frac{(\cos^2 + 2\cos + 4)}{-(\cos - 2)} \longrightarrow$

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$\left| f'_-(\frac{3}{4}) - f'_+(\frac{3}{4}) \right| = 2\sqrt{6}$

$(-4x+3)(\sqrt{ax}) - (4x-3)(\sqrt{ax}) \xrightarrow{\text{مشتق}} \left|(-4\sqrt{ax}) - (4\sqrt{ax})\right| = \overset{+2}{-8}\sqrt{a \times \frac{3}{4}} = 2\sqrt{6}$ ۳

$\sqrt{a \times \frac{3}{4}} = \sqrt{6} \times \frac{1}{\sqrt{16}}$ $\quad \frac{3a}{4} = \frac{6}{4 \cdot 16}$ $\quad 12a = 6$ $\boxed{a = \frac{1}{2}}$

۲

$f(4) = -4a+2$ $\quad f'(4) = a$ $\quad -4a+2+a = -1$ $\quad -3a = -3$

$a = 1$

$\boxed{\overline{f'(4)} = a = 1}$

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$7y - x = 5 \longrightarrow 7y = x+5 \quad y = \frac{x}{7} + \frac{5}{7}$

$\frac{x+5}{7} = \frac{ax-1}{3x+1}$ ، $\frac{1}{7} = \frac{a+3}{(3x+1)^2}$

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$f(x) = x^۳ + ax - b$ $\quad f(۲) = ۰ \quad ۸ + ۲a - b = ۰ \quad ۲a - b = -۸$

$f'(۲) = ۰ \quad ۳x^۲ + a \to ۱۲ + a = ۰ \quad a = -۱۲$

$-۲۴ - b = -۸ \to -۱۶ = b$

$b - a = (-۱۶) - (-۱۲) = \boxed{-۴}$ $+۱۲$

۲

ب

$f(x) = \dfrac{\sqrt{x}+1}{\sqrt{x}-1} \longrightarrow \sqrt{x} = -\dfrac{-y-1}{y-1} = \dfrac{y+1}{y-1} \longrightarrow x = \left(\dfrac{y+1}{y-1}\right)^۲$

$f^{-1}(x) = \left(\dfrac{x+1}{x-1}\right)^۲ \quad (f^{-1}(x))' = ۲\left(\dfrac{x+1}{x-1}\right)\left(\dfrac{(1)(x-1)-(1)(x+1)}{(x-1)^۲}\right) = ۲ \times ۳ \times \left(\dfrac{-۲}{۱}\right)$ ۴

$\boxed{-۱۲}$

ب

$f(x) = (x^۲+1)^۳(ax+1) \qquad -۱۱ = \dfrac{(۸)(1-a)-(1)(1)}{(۰)-(-1)} = ۸ - ۸a - ۱ = \dfrac{۷ - ۸a}{۱}$

$۷ - ۸a = -۱۱ \quad ۱۸ = ۸a \quad a = \dfrac{۱۸}{۸} = \dfrac{۹}{۴} \qquad -۲a = -۲ \times \dfrac{۹}{۴} = -\dfrac{۹}{۲}$

$f'(x) = (۳)(۲x)(x^۲+1)^۲\left(\dfrac{۹}{۴}x+1\right) + \left(\dfrac{۹}{۴}\right)(x^۲+1)^۳ \xrightarrow{x = -\frac{۹}{۲}}$

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$y = \sin x \times \cos ۲x \qquad \dfrac{\sin x \times \cos ۲x}{-\cos ۲x} = -\sin x$

$y = \sin^۴ x - \cos^۴ x = \sin^۲ - \cos^۲ x = -\cos ۲x$

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$-\sin\dfrac{\pi}{۲} - \left(-\sin\dfrac{\pi}{۴}\right) =$

$-۱ + \dfrac{\sqrt{۲}}{۲} = \dfrac{\sqrt{۲}}{۲} - ۱ = \boxed{\dfrac{\sqrt{۲}-۲}{۲}}$

$$g(x)= f'(\tan^2 x+\sqrt{2}\cos x)\times(2\tan x(1+\tan^2 x)-\sqrt{2}\sin x)$$ ۱

$$g'(\tfrac{\pi}{4})= f'(1+\sqrt{2}\times\tfrac{\sqrt{2}}{2})\times 2\times1\times(1+1^2)-(\sqrt{2}\times\tfrac{\sqrt{2}}{2})$$

$$\underbrace{\cancel{g'(\tfrac{\pi}{4})}}_{\sqrt{3}}= f'(2)\times\underbrace{[\cancel{2(2)}-1]}_{3}\longrightarrow f'(2)=\frac{\sqrt{3}}{3}$$

$$f-2g(\tfrac{7\pi}{4})=\frac{(2+\cancel{\cos x})(\cos^2 x-2\cos x+4)}{(2-\cos x)(2+\cancel{\cos x})}-\frac{4}{2-\cos x}=\frac{\overset{\cos x(\cancel{\cos x}-2)}{\cos^2 x\cancel{-2\cos x}}}{2-\cancel{\cos x}}=-\cos x$$ ۲

$$(f-2g)'=-(\cos x)'=+\sin x\longrightarrow \sin(\tfrac{7\pi}{4})=\boxed{-\frac{1}{2}}$$

$$y=-ax+2\longrightarrow f(4)=-4a+2\ ,\ f'(4)=-a$$ ۴

$$-4a+2-a=-1\to -5a=-3\longrightarrow a=+\frac{3}{5}\longrightarrow \boxed{f'(4)}=\boxed{-\frac{3}{5}}\overset{?}{=}-a$$

$$\frac{ax-1}{3x+1}=\frac{x}{7}+\frac{5}{7}\longrightarrow 7ax-7=3x^2+x+15x+5$$ ۵

$$3x^2+x(16-7a)+12=0\xrightarrow{\Delta=0}(16-7a)^2-12^2=(\underset{\boxed{a=\frac{4}{7}}}{16-7a-12})(\underset{\boxed{a=4}}{16-7a+12})$$

$$\boxed{a=4}\longrightarrow 3x^2-12x+12\longrightarrow \boxed{x=2}\ \checkmark$$

$a=\frac{4}{7}\longrightarrow 3x^2+12x+12\longrightarrow x=-2$ ✗ ناحیه اول

$$\lim_{x\to0}\frac{\sin^n(x^2)\times 2x\sin^{n-1}(x^2)\times\cos x^2}{(2\sin^2\frac{x}{2})^m}\overset{\text{هم ارزی}}{\leadsto}\lim_{x\to0}\frac{(x^2)^n\times n\times(x^2)^{n-1}\times2x}{(2(\frac{x}{2})^2)^m}=\frac{(x)^{2n+2n-2+1}\times 2n}{(x)^{2m}\times\frac{1}{2}}$$ ۷

$$=\frac{2nx^{4n-1}}{\frac{1}{2^m}\times x^{2m}}=2^{m+1}\times n\times x^{4n-2m-1}\rightsquigarrow 4n-2m-1=0\to n=\frac{2m+1}{4}$$

$$\rightsquigarrow 2^{m+1}\times\frac{2m+1}{4}=32\sqrt{2}\longrightarrow m=\frac{7}{2}\ ,\ n=2\to \boxed{2m+n=9}$$

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$$\frac{f(0)-f(-1)}{0-(-1)} = \frac{1-8(-a+1)}{1} = 8a-7=-11 \to 8a=-4 \to \boxed{a=-\frac{1}{2}}$$

$$f'(x)= 3(x^2+1)^2(2x)(ax+1) + a(x^2+1)^3 \xrightarrow{-2a=1} f'(1)= 3(2)^2(2)(\frac{1}{2}) + -\frac{1}{2}(2)^3$$

$$f'(1) = 12 - 4 = 8$$

۱۰

$$x=\frac{\pi}{2} \to \sin\frac{\pi}{4}\cos\frac{\pi}{4}=0$$

$$x=\frac{\pi}{2} \to \sin\frac{\pi}{2}\cos\pi=-1 \longrightarrow \frac{-1-0}{\frac{\pi}{2}} = \boxed{-\frac{4}{\pi} = \frac{\Delta y}{\Delta x}} \quad 1$$

$$\sin^4 x-\cos^4 x = (\sin^2 x+\cos^2 x)(\sin^2 x-\cos^2 x) = -\cos 2x$$

$$x=\frac{\pi}{2} \to -\cos\frac{\pi}{2}=0$$

$$x=\frac{\pi}{2} \to -\cos\pi=1 \longrightarrow \frac{1-0}{\frac{\pi}{2}} = \boxed{\frac{4}{\pi} = \frac{\Delta y}{\Delta x}} \quad 2 \longrightarrow \frac{-\frac{4}{\pi}}{\frac{4}{\pi}} = -1$$