صالح مثانی

به نام آنکه جان را فکرت آموخت

دوازدهم - تجربی B

تکلیف شماره ۳

①

$$g'(x) = \left(2\tan x(1+\tan^2 x) - \sqrt{2}\sin x\right) f'\left(\tan^2 x + \sqrt{2}\cos x\right) \to$$

$$g'\left(\frac{\pi}{4}\right) = f'(2) \times (2\times 2 - 1) = \frac{\sqrt{3}}{3}$$

②

$$f(x) = \frac{(\cos x + 2)(\cos^2 x + 4 - 2\cos x)}{(2-\cos x)(\cos x + 2)} = \frac{\cos^2 x - 2\cos x + 4}{2 - \cos x} \to f - 2g = \frac{\cos^2 x - 2\cos x}{2 - \cos x}$$

$$= \frac{-\cos x(2 - \cos x)}{2 - \cos x} = -\cos x \to f'(x) - 2g'(x) = \sin x \to f'\left(\frac{7\pi}{6}\right) - 2g'\left(\frac{7\pi}{6}\right) = \sin\frac{7\pi}{6} = -\frac{1}{2}$$

③

$$f(x)\begin{cases} x \to \frac{3}{4}^{+} \to (4x-3)\sqrt{ax} \to f'\left(\frac{3}{4}^{+}\right) = 4\left(\sqrt{\frac{3a}{2}}\right) + \underbrace{(0^{+})\frac{a}{2\sqrt{\frac{3a}{2}}}}_{=0} \\ x \to \frac{3}{4}^{-} \to (-4x+3)\sqrt{ax} \to f'\left(\frac{3}{4}^{-}\right) = -4\left(\sqrt{\frac{3a}{2}}\right) + \underbrace{(0^{-})\frac{a}{2\sqrt{\frac{3a}{2}}}}_{=0} \end{cases}$$

$$\to f'_{+}\left(\frac{3}{4}\right) - f'_{-}\left(\frac{3}{4}\right) = 2\sqrt{6} = 8\sqrt{\frac{3a}{2}} = 4\sqrt{3a} \to \sqrt{6} = 2\sqrt{3a} \to a = \frac{1}{2}$$

④

$$\begin{cases} f(4) = 2 - 4a \\ f'(4) = -a \end{cases} \Rightarrow 2 - 4a - a = -1 \to a = \frac{3}{5} \Rightarrow f'(4) = -\frac{3}{5}$$

5

$7y = x+5 \xrightarrow{x=y} 6x = 5 \to x = \frac{5}{6} \Rightarrow y' = \frac{a+3}{(3x+1)^2} = \frac{1}{7} \xrightarrow{x=\frac{5}{6}} \frac{a+3}{\frac{7}{2}\times\frac{7}{2}} = \frac{1}{7} \to a = -\frac{5}{4}$

$\hookrightarrow m = \frac{1}{7}$

6

$f'(2) = 0 \to 3x^2 + a = 0 \to 12 + a = 0 \to a = -12$

$f(2) = 0 \to +8 - 24 - b = 0 \to b = -16$ $\Rightarrow b - a = \frac{-4}{2}$

7

$f'(x) = 2nx \sin^{n-1}(x^2) \cdot \cos(x^2) \Rightarrow \lim_{x\to 0} \frac{f(x) \cdot f'(x)}{(1-\cos x)^m} = \lim_{x\to 0} \frac{\sin^n(x^2) \cdot 2nx \cos(x^2) \sin^{n-1}(x^2)}{\underbrace{(1-\cos x)^m}_{1-\frac{x^2}{2}}}$

هم‌ارزی $= \lim_{x\to 0} \frac{2nx \sin^{2n-1}(x^2)}{(\frac{x^2}{2})^m} = \lim_{x\to 0} \frac{2nx^{4n-1}}{\frac{x^{2m}}{2^m}} = \lim_{x\to 0} \frac{2^{m+1} \times n \times x^{4n-1}}{x^{2m}} = 32\sqrt{2} \to$

$2m = 4n - 1 \quad -m = 2n - 1 \to n = 2,\ m = \frac{7}{2} \to 2m + n = \frac{9}{2}$

8

$f(x) = 2 \Rightarrow 2 = \frac{\sqrt{x}+1}{\sqrt{x}-1} \to x = 9 \to (f^{-1}(2))' = \frac{1}{f'(9)} \Rightarrow f'(x) = \frac{\frac{1}{2\sqrt{x}}(\sqrt{x}-1) - \frac{1}{2\sqrt{x}}(\sqrt{x}+1)}{(\sqrt{x}-1)^2}$

$= \frac{\frac{1}{6}(2-4)}{4} = -\frac{1}{12} \to (f^{-1}(2))' = -12$

9

$\frac{f(0) - f(-1)}{0-(-1)} = 11 = \frac{1 + 8a - 8}{1} \to 8a - 7 = -11 \to a = -\frac{1}{2}$

$f'(x) = 3(x^2+1)(2x)(-\frac{1}{2}x + 1) - \frac{1}{2}(x^2+1)^3 \Rightarrow f'(1) = \frac{8}{2}$

10

$\frac{f(\frac{\pi}{2}) - f(\frac{\pi}{4})}{\frac{\pi}{2} - \frac{\pi}{4}}$ $\xrightarrow{\text{آهنگ متوسط}} f_1(x) = \frac{\sin\frac{\pi}{2}\cos\frac{\pi}{2} - \sin\frac{\pi}{4}\cos\pi}{\frac{\pi}{4}} = \frac{-1}{\frac{\pi}{4}}$

$\hookrightarrow f_2(x) = (\sin^2 - \cos^2)(\sin^2 + \cos^2) = \sin^2 - \cos^2 \xRightarrow{\text{آهنگ متوسط}} \frac{\sin^2\frac{\pi}{2} - \cos^2\frac{\pi}{2} - \sin^2\frac{\pi}{4} + \cos^2\frac{\pi}{4}}{\frac{\pi}{4}} =$

$\frac{1}{\frac{\pi}{4}} \leftarrow$ نسبت $= -1$