نام و نام خانوادگی ...امیریال توسلی... پاسخنامه تشریحی تکلیف شماره ...۲۳... کلاس ...دوازدهم B...

$g(x)=f(\tan^2 x+\sqrt{2}\cos x) \rightsquigarrow g'(x)=\left(2\tan x\,\frac{1}{\cos^2 x}-\sqrt{2}\sin x\right)f'(\tan^2 x+\sqrt{2}\cos x)$ $g'(\frac{\pi}{4})=(4-1)f'(1+1) \Rightarrow g'(\frac{\pi}{4})=3f'(2)=\frac{\sqrt{3}}{3} \rightsquigarrow f'(2)=\frac{\sqrt{3}}{3}$	۱
$f(x)=\frac{8+\cos^3 x}{4-\cos^2 x}=\frac{\cancel{(2+\cos x)}(4-2\cos x+\cos^2 x)}{\cancel{(2+\cos x)}(2-\cos x)}=\frac{4-2\cos x+\cos^2 x}{2-\cos x}$ $g(x)=\frac{2}{2-\cos x} \to f(x)-2g(x)=\frac{4-2\cos x+\cos^2 x-4}{2-\cos x}=\frac{\cos^2 x-2\cos x}{2-\cos x}$ $\to \frac{\cos x(\cos x-2)}{2-\cos x}=-\cos x \to h(x)=f(x)-2g(x) \to -\cos x \to h'(\frac{7\pi}{4})=\sin\frac{7\pi}{4}$ $-\frac{\sqrt{2}}{2}$	۲
$f(x)=\lvert 4x-3\rvert\sqrt{ax} \Rightarrow \begin{cases}(4x-3)\sqrt{ax} & x\geq\frac{3}{4}\\ (3-4x)\sqrt{ax} & x<\frac{3}{4}\end{cases}$ $\quad \lvert f'_+(\frac{3}{4})-f'_-(\frac{3}{4})\rvert=2\sqrt{6}$ $\Rightarrow ((8x-6)\sqrt{ax})' \rightsquigarrow 8\sqrt{ax}+\cancel{(8x-6)\frac{a}{2\sqrt{ax}}} \Rightarrow 8\sqrt{ax}$, $x\to\frac{3}{4}$ $\Rightarrow \cancel{8}^{4}\sqrt{ax}=\cancel{2}\sqrt{6} \rightsquigarrow \cancel{4}\sqrt{\frac{3a}{\cancel{4}}}=\sqrt{6} \Rightarrow 2\sqrt{\cancel{3}}\times\sqrt{a}=\sqrt{\cancel{3}}\times\sqrt{2} \Rightarrow \sqrt{a}=\frac{\sqrt{2}}{2} \rightsquigarrow a=\frac{1}{2}$	۳
$y+ax=2 \rightsquigarrow y=-ax+2$ $f(4)+f'(4)=-1 \quad \Rightarrow f(4)=-4a+2 \quad f'(4)=-a$ $\Rightarrow -4a+2-a=-1 \rightsquigarrow -5a=-3 \rightsquigarrow \boxed{a=\frac{3}{5}} \to \boxed{f'(4)=-a=-\frac{3}{5}}$	۴
$7y-x=5 \rightsquigarrow y=\frac{x+5}{7}$ $y=\frac{ax-1}{3x+1} \Rightarrow \frac{x+5}{7}=\frac{ax-1}{3x+1} \to 3x^2+16x+5=7ax-7$ $\Rightarrow 3x^2+(16-7a)x+12=0$ $\Delta=0 \to (16-7a)^2=144 \rightsquigarrow$ $16-7a=12 \Rightarrow 7a=4 \rightsquigarrow \cancel{a=\frac{4}{7}}$ $16-7a=-12 \Rightarrow 7a=28 \to a=4$	۵

$f(x) = x^3 + ax - b \longrightarrow f(2) = 0 \Rightarrow 8 - \cancel{12 \times 2}^{-24} - b = 0 \Rightarrow b = -16$

$f'(x) = 3x^2 + a$ $\longrightarrow b - a \Rightarrow -16 - (-12) = -4$ 6

$f'(2) = 0$

$\hookrightarrow 12 + a = 0 \longrightarrow a = -12$

$f(x) = \sin^n(x^2) \longrightarrow f'(x) = 2nx \cos(x^2) \times \sin^{n-1}(x^2)$

$\Rightarrow \lim \dfrac{f(x) \cdot f'(x)}{(1-\cos x)^m} \Rightarrow \sin^n(x^2) \cdot \sin^{n-1}(x^2) \cdot 2nx \cos(x^2) \Rightarrow \dfrac{\sin^{2(n-1)}(x^2) \cdot nx \cdot \sin(2x^2)}{(1-\cos x)^m}$ 7

$\Rightarrow$ (هم‌ارزی) $\Rightarrow \dfrac{(x^2)^{2(n-1)} \times nx \times 2x^2}{\left(\frac{x^2}{2}\right)^m} \Rightarrow \dfrac{x^{4n-1} \times 2n \times 2^m}{x^{2m}} = 32\sqrt{2}$ $\begin{cases} 4n-1 = 2m \\ \hookrightarrow m = \frac{4n-1}{2} \\ 2^{m+1} \times n = 2^{\frac{11}{2}} \end{cases}$

$\Rightarrow 2^{\frac{4n+1}{2}} \times n = 2^{\frac{11}{2}} \longrightarrow n = 2 \longrightarrow m = \frac{7}{2} \longrightarrow \boxed{2m + n \Rightarrow 9}$

$$f(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow f^{-1}(x)=\left(\frac{x+1}{x-1}\right)^{۲} \rightsquigarrow f'^{-1}(x)=۲\left(\frac{x+1}{x-1}\right)\left(\frac{\overset{-۲}{\cancel{(x-1)-(x+1)}}}{(x-1)^{2}}\right)$$

$$x=۲ \rightsquigarrow ۲(۳)\times -۲=-۱۲$$

سوال ۸

$\hookrightarrow \frac{۴(1+\frac{1}{۴})-14}{۴} \rightsquigarrow$

۹

$f(x)=(x^۲+1)^۳(ax+1) \Rightarrow f(0)=1$

$f(-1)=(1-a)(\cancel{۲})^۳ ۸ \rightsquigarrow \frac{1+۸(a-1)}{1}=-11$

$\rightsquigarrow 1+۸a-۸=-11 \Rightarrow a=-\frac{1}{۲} \longrightarrow ۲a=-1 \longrightarrow -۲a=1$

$f'(x)=۳\times۲x(x^۲+1)^۲(ax+1)+a(x^۲+1)^۳ \rightsquigarrow ۶\times\cancel{۴}^{۲}\times\frac{1}{\cancel{۲}}-۸\times\frac{1}{۲}=۱۲-۴=۸$

۱۰

$D=[\frac{\pi}{۴},\frac{\pi}{۲}] \quad y=\sin x\cos ۲x \Rightarrow \begin{cases} x=\frac{\pi}{۲} \Rightarrow 1\times-1=-1 \\ x=\frac{\pi}{۴} \Rightarrow \frac{\sqrt{۲}}{۲}\times 0=0 \end{cases} \rightsquigarrow \frac{-1}{\frac{\pi}{۴}} \rightarrow -\frac{۴}{\pi}$

تغییرات متوسط تابع ①

$y=\sin^۴x-\cos^۴x \rightsquigarrow (\sin^۲x-\cos^۲x)\underbrace{(\cancel{\sin^۲x+\cos^۲x})}_{1}$

$\Rightarrow \sin^۲x-\cos^۲x=-\cos۲x$

$\hookrightarrow -\cos۲x \begin{cases} x=\frac{\pi}{۲} \rightarrow 1 \\ x=\frac{\pi}{۴} \rightarrow 0 \end{cases} \rightsquigarrow \frac{1-0}{\frac{\pi}{۴}}=\frac{۴}{\pi}$

متوسط تغ ②

نسبت آنها: $\frac{①}{②}=\boxed{\frac{-\frac{۴}{\pi}}{\frac{۴}{\pi}}=-1}$