نام و نام خانوادگی ...امیر یل توسلی... پاسخنامه تشریحی تکلیف شماره ...۲۳... کلاس ...دوازدهم B...

۲۰ آفرین

$g(x) = f(\tan^2 x + \sqrt{2}\cos x) \rightarrow g'(x) = \left(2\tan x \cdot \frac{1}{\cos^2 x} - \sqrt{2}\sin x\right) f'(\tan^2 x + \sqrt{2}\cos x)$

$g'(\frac{\pi}{4}) = (4-1)\, f'(1+1) \Rightarrow g'(\frac{\pi}{4}) = f'(2) = \frac{\sqrt{3}}{3} \rightarrow f'(2) = \frac{\sqrt{3}}{3}$ ۲ ۱

$f(x) = \frac{8+\cos^3 x}{4-\cos^2 x} = \frac{\cancel{(2+\cos x)}(4-2\cos x+\cos^2 x)}{\cancel{(2+\cos x)}(2-\cos x)} = \frac{4-2\cos x+\cos^2 x}{2-\cos x}$ ۴

$g(x) = \frac{2}{2-\cos x} \rightarrow f(x)-2g(x) = \frac{4-2\cos x+\cos^2 x-4}{2-\cos x} = \frac{\cos^2 x-2\cos x}{2-\cos x}$ ۲

$\rightarrow \frac{\cos x(\cos x-2)}{2-\cos x} = -\cos x \rightarrow h(x) = f(x)-2g(x) \rightarrow -\cos x \rightarrow h'(\frac{7\pi}{4}) = \sin\frac{7\pi}{4}$ ۲

$f(x) = |4x-3|\sqrt{ax} \Rightarrow \begin{cases} (4x-3)\sqrt{ax} & x \geq \frac{3}{4} \\ (3-4x)\sqrt{ax} & x < \frac{3}{4} \end{cases} \quad \left|f'_+(\frac{3}{4}) - f'_-(\frac{3}{4})\right| = 2\sqrt{6}$

$\Rightarrow ((8x-6)\sqrt{ax})' \rightarrow 8\sqrt{ax} + \underbrace{(8x-6)}_{0}\frac{a}{2\sqrt{ax}} \Rightarrow 8\sqrt{ax}, \quad x \rightarrow \frac{3}{4}$ ۲ ۳

$\Rightarrow \cancel{8}^{4}\sqrt{ax} = \cancel{2}\sqrt{6} \rightarrow \cancel{4}\frac{\sqrt{3a}}{\cancel{2}} = \sqrt{6} \Rightarrow 2\sqrt{\cancel{3}} \times \sqrt{a} = \sqrt{\cancel{6}} \times \sqrt{2} \Rightarrow \sqrt{a} = \frac{\sqrt{2}}{2} \rightarrow a = \frac{1}{2}$

$y + ax = 2 \rightarrow y = -ax+2$

$f(4) + f'(4) = -1 \quad \Rightarrow f(4) = -4a+2 \quad f'(4) = -a$

$\Rightarrow -4a+2-a = -1 \rightarrow -5a = -3 \rightarrow \boxed{a = \frac{3}{5}} \rightarrow \boxed{f'(4) = -a = -\frac{3}{5}}$ ۴

۲

$7y - x = 5 \rightarrow y = \frac{x+5}{7}$

$y = \frac{ax-1}{3x+1} \Rightarrow \frac{x+5}{7} = \frac{ax-1}{3x+1} \rightarrow 3x^2+16x+5 = 7ax-7 \Rightarrow 3x^2+(16-7a)x+12=0$

$\Delta=0 \rightarrow (16-7a)^2 = 144 \rightarrow 16-7a=12 \Rightarrow 7a=4 \rightarrow a=\frac{4}{7}$ ۲ ۵

$16-7a=-12 \Rightarrow 7a=28 \rightarrow a=4$

$$f(x) = x^3 + ax - b \longrightarrow f(2) = 0 \Rightarrow 8 - \overset{-24}{\cancel{12 \times 2}} - b = 0 \Rightarrow b = -16$$

$$f'(x) = 3x^2 + a$$

$$f'(2) = 0 \qquad \longrightarrow b - a \Rightarrow -16 - (-12) = -4$$

$$\hookrightarrow 12 + a = 0 \longrightarrow a = -12$$

2 6

$$f(x) = \sin^n(x^2) \longrightarrow f'(x) = 2nx\cos(x^2) \times \sin^{n-1}(x^2)$$

$$\Rightarrow \frac{f(x) \cdot f'(x)}{(1-\cos x)^m} \Rightarrow \sin^n(x^2) \cdot \sin^{n-1}(x^2) \cdot 2nx\cos(x^2) \Rightarrow \frac{\sin^{2(n-1)}(x^2) \cdot nx \cdot \sin(2x^2)}{(1-\cos x)^m}$$

$$\Rightarrow ([illegible]) \Rightarrow \frac{(x^2)^{2(n-1)} \times nx \times 2x^2}{\left(\frac{x^2}{2}\right)^m} \Rightarrow \frac{x^{4n-1} \times 2n \times 2^m}{x^{2m}} = 32\sqrt{2} \quad \begin{cases} 4n-1 = 2m \to m = \frac{4n-1}{2} \\ 2^{m+1} \times n = 2^{\frac{11}{2}} \end{cases}$$

2 7

$$\Rightarrow 2^{\frac{4n+1}{2}} \times n = 2^{\frac{11}{2}} \longrightarrow n = 2 \longrightarrow m = \frac{7}{2} \longrightarrow \boxed{2m + n \Rightarrow 9}$$

سوال ۸

$$f(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow f^{-1}(x)=\left(\frac{x+1}{x-1}\right)^{2} \rightsquigarrow f'^{-1}(x)=2\left(\frac{x+1}{x-1}\right)\left(\frac{\overset{-2}{\cancel{(x-1)-(x+1)}}}{(x-1)^{2}}\right)$$

$$x=2 \rightsquigarrow 2(3)\times -2 = -12$$

$\hookrightarrow \frac{2(1+\frac{1}{4})-14}{4} \rightsquigarrow$

9

$f(x) = (x^2+1)^3(ax+1) \Rightarrow f(0) = 1$

$f(-1) = (1-a)(\cancel{2})^3 8 \rightarrow \frac{1+8(a-1)}{1} = -11$

$\rightarrow 1+8a-8 = -11 \Rightarrow a = -\frac{1}{2} \longrightarrow 2a = -1 \longrightarrow -2a = 1$

۲

$f'(x) = 3\times 2x(x^2+1)^2(ax+1) + a(x^2+1)^3 \rightsquigarrow 6 \times 4 \times \frac{1}{2} - 8 \times \frac{1}{2} = 12 - 4 = 8$

10

$D = [\frac{\pi}{4}, \frac{\pi}{2}] \quad y = \sin x \cos 2x \Rightarrow \begin{cases} x = \frac{\pi}{2} \Rightarrow 1 \times -1 = -1 \\ x = \frac{\pi}{4} \Rightarrow \frac{\sqrt{2}}{2} \times 0 = 0 \end{cases} \rightsquigarrow \frac{-1}{\frac{\pi}{4}} \rightarrow -\frac{4}{\pi}$

$y = \sin^4 x - \cos^4 x \rightarrow (\sin^2 x - \cos^2 x)(\cancel{\sin^2 x + \cos^2 x})$ ① تابع متوسط تغییرات ←

$\Rightarrow \sin^2 x - \cos^2 x = -\cos 2x$

$\hookrightarrow -\cos 2x \begin{cases} x = \frac{\pi}{2} \rightarrow 1 \\ x = \frac{\pi}{4} \rightarrow 0 \end{cases} \rightsquigarrow \frac{1-0}{\frac{\pi}{4}} = \frac{4}{\pi} \rightsquigarrow$ متوسط تغ ② نسبت آنا: $\rightarrow \frac{①}{②} = \boxed{\frac{-\frac{4}{\pi}}{\frac{4}{\pi}} = -1}$