مهدی کردسیچانی – تکلیف ۲۳

①

$g'(x) = \left(2\times \frac{1}{\cos^2 x}\times \tan x - \sqrt{2}\cos x\right) f'(\tan^2 x+\sqrt{2}\cos x)$

$g'(\frac{\pi}{4}) = (2\times 2\times 1 - 1) f'(2) = \sqrt{3} = 3f'(2)=\sqrt{3} \Rightarrow f'(2) = \frac{\sqrt{3}}{3}$

②

$f(x) = \frac{(2+\cos x)(4+\cos^2 x - 2\cos x)}{(2+\cos x)(2-\cos x)} = \frac{(2-\cos x)^2}{2-\cos x} = 2-\cos x \Rightarrow f' = \sin x$

$f'(\frac{7\pi}{6}) = \frac{-1}{2}$ $\qquad g'(x) = \frac{-2(\sin x)}{(2-\cos x)^2} \Rightarrow g'(\frac{7\pi}{6}) =$

③

$x>\frac{3}{4} \Rightarrow f(x) = (4x-3)\sqrt{ax} \to f'(x) = 4\sqrt{ax} \Rightarrow f'_+(\frac{3}{4}) = 2\sqrt{3a}$

$x<\frac{3}{4} \Rightarrow f(x) = -4\sqrt{ax} \Rightarrow f'_-(\frac{3}{4}) = -2\sqrt{3a}$

$f'_+(\frac{3}{4}) - f'_-(\frac{3}{4}) = 2\sqrt{6} = 4\sqrt{3a} \Rightarrow a=\frac{1}{2}$

④

$f'(4) = -a \quad , \quad f(4) = 2-4a \Rightarrow 2-5a=-1 \Leftarrow y=-ax+2$

$a=\frac{3}{5} \Rightarrow f'(4) = \frac{-3}{5}$

⑤

$\frac{ax-1}{3x+1} = \frac{x+d}{7}$ (مقدار برابر) $\qquad \frac{1}{7} = \frac{a+3}{(3x+1)^2}$ (مشتق برابر) $\Leftrightarrow \qquad y=\frac{1}{7}x+\frac{d}{7}$

با توجه به این دو رابطه x را ۲ بدست می‌آوریم و در نتیجه $a=4$ می‌شود

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$f'(x) = 1 \Rightarrow 3x^2 + a = f'(x) \Rightarrow 12 + a = 1 \Rightarrow a = -11$ (۶)

$f(2) = 2 \Rightarrow 8 + 2a - b = 8 - 22 - b = 2 \Rightarrow b = -16$

$b - a = -16 + 11 = -5$

$f'(x) = 2n \sin^{n-1}(x^2) \cdot \cos x^2 \Rightarrow \lim_{x \to 0} \frac{\sin^n(x^2) \times 2nx \cos x^2 \sin^{n-1} x^2}{(1 - \cos x)^m}$ (۷)

$= \lim \frac{2nx \sin^{2n-1} x^2}{(\frac{x^2}{2})^m} = \lim_{x \to 0} \frac{2nx^{4n-1}}{\frac{x^{2m}}{2^m}} = \lim \frac{2^{m+1} \times n \times x^{4n-1}}{x^{2m}} = 32\sqrt{2}$

$\Rightarrow 2m = 4n - 1 \Rightarrow m = 2n - \frac{1}{2} \Rightarrow n = 2$

$2m = 5 \Rightarrow 2m + n = 9$ (۹)

$x = \frac{\sqrt{y} + 1}{\sqrt{y} - 1} \Rightarrow f^{-1}(x) = \left(\frac{x+1}{x-1}\right)^2 \Rightarrow f^{-1'}(x) = 2 \times \frac{x+1}{x-1} \times \frac{-2}{(x-1)^2}$ (۱۰)

$\Rightarrow f^{-1'}(2) = 2 \times 3 \times -2 = -12$

$m_{-1} = -11 = \frac{1 + 8a - 8}{1} \Rightarrow 8a - 7 = -11 \Rightarrow a = \frac{-1}{2}$ (۹)

$\Rightarrow x = 1 \rightarrow f'(x) = 3(2x)(x^2+1)^2(ax+1) + a(x^2+1)^3$

$\Rightarrow f'(1) = 9 \times 4 \times \frac{1}{2} + \frac{-1}{2} \times 8 = 8$

$x = \frac{\pi}{2} \rightarrow y = -1$
$x = \frac{\pi}{4} \rightarrow y = 0$
$\Big\} m = \frac{-1}{\frac{\pi}{4}} = \frac{-4}{\pi}$

$\Rightarrow \frac{\frac{-4}{\pi}}{\frac{4}{\pi}} = -1$ (۱۰)

$y = (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) = -\cos 2x \Rightarrow$
$x = \frac{\pi}{2} \rightarrow y = 1$
$x = \frac{\pi}{4} \rightarrow y = 0$
$\Big\} m' = \frac{1}{\frac{\pi}{4}} = \frac{4}{\pi}$

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