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تکلیف ۲۳ ۱۲ تیر B

۱.

$$g(x) = f(\overbrace{\tan^2 x + \sqrt{2}\cos x}^{h(x)}) = f(h(x)) = f\circ h(x)$$

$$\Rightarrow g'(x) = (f\circ h)'(x) = h'(x)\, f'(h(x)) = (\tan^2 x + \sqrt{2}\cos x)' \times f'(\tan^2 x + \sqrt{2}\cos x)$$

$$\Rightarrow g'(\tfrac{\pi}{4}) = (\tan^2 x + \sqrt{2}\cos x)' \times f'(\underbrace{\tan^2 \tfrac{\pi}{4} + \sqrt{2}\cos \tfrac{\pi}{4}}_{2})$$

$$\Rightarrow g'(\tfrac{\pi}{4}) = \big((2\tan x)(1+\tan^2 x) - \sqrt{2}\sin x\big) \times f'(2) = \sqrt{3}$$

$$\Rightarrow g'(\tfrac{\pi}{4}) = \sqrt{3} = (2\times 2 - 1)\times f'(2) \Rightarrow f'(2) = \frac{\sqrt{3}}{3}$$

۲.

$$f(x) = \frac{(2+\cos x)(4+\cos^2 x - 2\cos x)}{(2-\cos x)(2+\cos x)} = \frac{4+\cos^2 x - 2\cos x}{2-\cos x}$$

$$g(x) = \frac{2}{2-\cos x} \Rightarrow 2g(x) = \frac{4}{2-\cos x}$$

$$\Rightarrow f(x) - 2g(x) = \frac{4+\cos^2 x - 2\cos x}{2-\cos x} - \frac{4}{2-\cos x} = \frac{\cos x(\cos x - 2)}{2-\cos x}$$

$$= -\cos x \Rightarrow (f(x) - 2g(x))' = f'(x) - 2g'(x) = \sin x$$

$$\Rightarrow f'(\tfrac{7\pi}{6}) - 2g'(\tfrac{7\pi}{6}) = \sin\tfrac{7\pi}{6} = -\tfrac{1}{2}$$

۳.

$$f'_+(\tfrac{3}{4}) = \lim_{x\to \frac{3}{4}^+} \frac{|4x-3|\sqrt{ax} - 0}{x-\frac{3}{4}} = \frac{(4x-3)\sqrt{ax}}{x-\frac{3}{4}} = 4\sqrt{\tfrac{3}{4}a}$$

$$f'_-(\tfrac{3}{4}) = \lim_{x\to \frac{3}{4}^-} \frac{|4x-3|\sqrt{ax} - 0}{x-\frac{3}{4}} = \frac{-(4x-3)\sqrt{ax}}{x-\frac{3}{4}} = -4\sqrt{\tfrac{3}{4}a}$$

$$\Rightarrow 4\sqrt{\frac{3}{4}a} - \left(-4\sqrt{\frac{3}{4}a}\right) = 8\sqrt{\frac{3}{4}a} = 2\sqrt{6} \Rightarrow 64\times\frac{3}{4}a = 6\times2\times2$$

$$\Rightarrow a = \frac{4\times6\times2\times2}{3\times64} = \frac{1}{2}$$

۴-

$$y = -ax + 2 \xrightarrow{x=4} y = f(4) = -4a + 2$$

$$f'(4) = -a \qquad f'(4) = -a$$

$$f(4) + f'(4) = -4a + 2 - a = -5a + 2 = -1 \Rightarrow a = \frac{3}{5} \Rightarrow f'(4) = -a = -\frac{3}{5}$$

۵-

$$7y - x = 5 \Rightarrow y = +\frac{1}{7}x + \frac{5}{7}$$

چون منحنی ها مماس شده اند پس معادله تلاقی آنها ریشه مضاعف دارد:
($\Delta = 0$)

$$\frac{ax-1}{3x+1} = \frac{1}{7}x + \frac{5}{7} \Rightarrow \frac{3}{7}x^2 + \frac{16}{7}x + \frac{5}{7} = ax - 1$$

$$\Rightarrow \frac{3}{7}x^2 + \left(\frac{16}{7} - a\right)x + \frac{12}{7} = 0 \Rightarrow 3x^2 + (16 - 7a)x + 12 = 0$$

$$\Rightarrow \Delta = 0 \Rightarrow (16 - 7a)^2 - 4(3)(12) = 0 \Rightarrow (16 - 7a)^2 = 12\times4\times3 = 9\times16$$

$$\Rightarrow \begin{cases} 16 - 7a = 3\times4 = 12 \Rightarrow a = \frac{4}{7} \Rightarrow 3x^2 + 12x + 12 = 0 \Rightarrow x = -2 \quad \text{غ ق ق} \\ 16 - 7a = -3\times4 = -12 \Rightarrow a = 4 \Rightarrow 3x^2 - 12x + 12 = 0 \Rightarrow x = \frac{-b}{2a} = 2 \end{cases}$$

$\Rightarrow a = 4$ است.

۶- در نقطه ۲ بر محور طول ها مماس است پس:

① $f(2) = 0$ ② $f'(2) = 0$

① $\Rightarrow f(x) = x^3 + ax - b \xrightarrow{x=2} 8 + 2a - b = 0$

② $\Rightarrow f'(x) = 3x^2 + a \Rightarrow 12 + a = 0 \Rightarrow a = -12$

$$\begin{cases} 8 - 24 - b = 0 \\ \Rightarrow b = -16 \\ \Rightarrow b - a = -16 - (-12) = -4 \end{cases}$$

۷-

$f(x)=\sin^n(x^2) \Rightarrow f'(x)=n\sin^{n-1}(x^2)\cos(x^2)\times 2x$

$$\lim_{x\to 0}\frac{f(x)\,f'(x)}{(1-\cos x)^m}=\frac{\sin^n(x^2)\times n\sin^{n-1}(x^2)\times\cos(x^2)\times 2x}{(1-\cos x)^m}$$

هم ارزی

$$\lim_{x\to 0}\frac{x^{2n}\times n x^{2n-2}\times(1-\frac{x^4}{2})\times 2x}{\frac{x^{2m}}{2^m}}=\frac{n x^{4n-1}\times 2^{m+1}\times(1-\frac{x^4}{2})}{x^{2m}}=32\sqrt{2}$$

$\Rightarrow \begin{cases} 4n-1=2m \Rightarrow m=2n-\frac{1}{2} \\ \lim_{x\to 0}\frac{n x^{4n-1}\times 2^{m+1}\times(1-\frac{x^4}{2})}{x^{4n}}=\frac{2^{m+1}\times n}{1}=2^{2n+\frac{1}{2}}\times n=32\sqrt{2} \end{cases}$

$\Rightarrow n=2 \Rightarrow m=3.5$

$\Rightarrow 2m+n=7+2=9$

۸-

$y=\frac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow \sqrt{x}=\frac{y+1}{y-1} \Rightarrow x=\left(\frac{y+1}{y-1}\right)^2$

$\Rightarrow f^{-1}(x)=\left(\frac{x+1}{x-1}\right)^2 \Rightarrow [f^{-1}(x)]'=2\left(\frac{x+1}{x-1}\right)\times\left(\frac{-2}{(x-1)^2}\right)$

$\xRightarrow{x=2} [f^{-1}(x)]'=2\times 3\times -2=-12$

۹-

$$\text{آهنگ تغییر متوسط}=\frac{f(a)-f(b)}{a-b}=\frac{8(-a+1)-1}{-1-0}=8a-7=-11 \Rightarrow a=-\frac{1}{2}$$

$f(x)=(x^2+1)^3(-\frac{1}{2}x+1) \Rightarrow f'(1)=3(x^2+1)^2\times 2x(-\frac{1}{2}x+1)+(-\frac{1}{2})(x^2+1)^3$

$=3(2)^2\times 2(\frac{1}{2})+(-\frac{1}{2})(8)$

$a=-2a=1$

$=12+(-4)=8$

۱۰-

آهنگ متوسط تغییر $= \dfrac{f(a)-f(b)}{a-b} \Rightarrow y=\sin x\cos^2 x \quad [\frac{\pi}{4},\frac{\pi}{2}] \rightarrow$ آهنگ متوسط $= \dfrac{\sin\frac{\pi}{4}\cos\frac{\pi}{2}-\sin\frac{\pi}{2}\cos\ldots}{\frac{\pi}{4}-\frac{\pi}{2}}$

$$= \frac{0-(-1)}{-\frac{\pi}{4}} = -\frac{4}{\pi} \quad \mathrm{I}$$

$$y=\sin^4 x-\cos^4 x=\underbrace{(\sin^2 x-\cos^2 x)}_{-\cos 2x}\underbrace{(\cos^2 x+\sin^2 x)}_{1}=-\cos 2x$$

آهنگ متوسط تغییر $= \dfrac{-\cos\frac{\pi}{2}-(-\cos\pi)}{\frac{\pi}{4}-\frac{\pi}{2}} = \dfrac{0-(-(-1))}{-\frac{\pi}{4}} = \dfrac{-1}{-\frac{\pi}{4}} = \dfrac{4}{\pi} \quad \mathrm{II}$

$$\mathrm{I},\mathrm{II} \Rightarrow \frac{-\frac{4}{\pi}}{\frac{4}{\pi}} = -1$$