به نام خدا
ماهان نژادکرد

۲۰ آفرین

تکلیف ۲۳   ۱۲ پسر B

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$$g(x) = f(\overbrace{\tan^2 x + \sqrt{2}\cos x}^{h(x)}) = f(h(x)) = f\circ h(x)$$

$$\Rightarrow g'(x) = (f\circ h)'(x) = h'(x)\, f'(h(x)) = (\tan^2 x + \sqrt{2}\cos x)' \times f'(\tan^2 x + \sqrt{2}\cos x)$$

$$\Rightarrow g'(\tfrac{\pi}{4}) = (\tan^2 x + \sqrt{2}\cos x)' \times f'(\underbrace{\tan^2 \tfrac{\pi}{4} + \sqrt{2}\cos \tfrac{\pi}{4}}_{2})$$

$$\Rightarrow g'(\tfrac{\pi}{4}) = ((2\tan x)(1+\tan^2 x) - \sqrt{2}\sin x) \times f'(2) = \sqrt{3}$$

$$\Rightarrow g'(\tfrac{\pi}{4}) = \sqrt{3} = (2\times 2 - 1)\times f'(2) \Rightarrow f'(2) = \frac{\sqrt{3}}{3}$$

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$$f(x) = \frac{(2+\cos x)(4+\cos^2 x - 2\cos x)}{(2-\cos x)(2+\cos x)} = \frac{4+\cos^2 x - 2\cos x}{2-\cos x}$$

$$g(x) = \frac{2}{2-\cos x} \Rightarrow 2g(x) = \frac{4}{2-\cos x}$$

$$\Rightarrow f(x) - 2g(x) = \frac{4+\cos^2 x - 2\cos x}{2-\cos x} - \frac{4}{2-\cos x} = \frac{\cos x(\cos x - 2)}{2-\cos x}$$

$$= -\cos x \Rightarrow (f(x) - 2g(x))' = f'(x) - 2g'(x) = \sin x$$

$$\Rightarrow f'(\tfrac{7\pi}{6}) - 2g'(\tfrac{7\pi}{6}) = \sin\tfrac{7\pi}{6} = -\frac{1}{2}$$

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$$f'_+(\tfrac{3}{4}) = \lim_{x\to \frac{3}{4}^+} \frac{|4x-3|\sqrt{ax} - 0}{x-\frac{3}{4}} = \frac{(4x-3)\sqrt{ax}}{x-\frac{3}{4}} = 4\sqrt{\tfrac{3}{4}a}$$

$$f'_-(\tfrac{3}{4}) = \lim_{x\to \frac{3}{4}^-} \frac{|4x-3|\sqrt{ax}}{x-\frac{3}{4}} = \frac{-(4x-3)\sqrt{ax}}{x-\frac{3}{4}} = -4\sqrt{\tfrac{3}{4}a}$$

$\Rightarrow 4\sqrt{\frac{3}{2}a} - (-4\sqrt{\frac{3}{2}a}) = 8\sqrt{\frac{3}{2}a} = 2\sqrt{6} \Rightarrow 64 \times \frac{3}{2}a = 6\times2\times2$

$\Rightarrow a = \frac{2\times6\times2\times2}{3\times64} = \frac{1}{4}$ ✓ ۲

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$y = -ax + 2 \xrightarrow{x=4} y = f(4) = -4a + 2$

$f'(4) = -a \qquad f'(4) = -a$

$f(4) + f'(4) = -4a + 2 - a = -5a + 2 = -1 \Rightarrow a = \frac{3}{5} \Rightarrow f'(4) = -a = -\frac{3}{5}$ ۲

۵ -

$7y - x = 5 \Rightarrow y = +\frac{1}{7}x + \frac{5}{7}$

چون منحنی مماس شده است پس معادله تلاقی آنها ریشه مضاعف دارد: $(\Delta = 0)$

خط بر

$\frac{ax-1}{3x+1} = \frac{1}{7}x + \frac{5}{7} \Rightarrow \frac{3}{7}x^2 + \frac{16}{7}x + \frac{5}{7} = ax - 1$

$\Rightarrow \frac{3}{7}x^2 + (\frac{16}{7} - a)x + \frac{12}{7} = 0 \Rightarrow 3x^2 + (16 - 7a)x + 12 = 0$

$\Rightarrow \Delta = 0 \Rightarrow (16 - 7a)^2 - 4(3)(12) = 0 \Rightarrow (16 - 7a)^2 = 12\times4\times3 = 9\times16$

$\Rightarrow \begin{cases} 16 - 7a = 3\times4 = 12 \Rightarrow a = \frac{4}{7} \Rightarrow 3x^2 + 12x + 12 = 0 \Rightarrow x = -2 \\ 16 - 7a = -3\times4 = -12 \Rightarrow a = 4 \Rightarrow 3x^2 - 12x + 12 = 0 \Rightarrow x = \frac{-b}{2a} = 2 \end{cases}$

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$\Rightarrow a = 4$ است

۶ - در نقطه ۲ بر محور طول ها مماس است پس:

① $f(2) = 0$ ② $f'(2) = 0$

① $\Rightarrow f(x) = x^3 + ax - b \xrightarrow{x=2} 8 + 2a - b = 0$

② $\Rightarrow f'(x) = 3x^2 + a \Rightarrow 12 + a = 0 \Rightarrow a = -12$

$\begin{cases} 8 - 24 - b = 0 \\ \Rightarrow b = -16 \\ \Rightarrow b - a = -16 - (-12) = -4 \end{cases}$ ۲

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$f(x) = \sin^n(x^2) \Rightarrow f'(x) = n\sin^{n-1}(x^2)\cos(x^2) \times 2x$

$$\lim_{x\to 0} \frac{f(x)\,f'(x)}{(1-\cos x)^m} = \frac{\sin^n(x^2) \times n\sin^{n-1}(x^2) \times \cos(x^2) \times 2x}{(1-\cos x)^m}$$

هم ارزی

$$\lim_{x\to 0} \frac{x^{2n} \times n x^{2n-2} \times (1-\frac{x^4}{2}) \times 2x}{\frac{x^{2m}}{2^m}} = \frac{n x^{4n-1} \times 2^{m+1} \times (1-\frac{x^4}{2})}{x^{2m}} = 32\sqrt{2}$$

$\Rightarrow 4n-1 = 2m \Rightarrow m = 2n - \frac{1}{2}$

$$\lim_{x\to 0} \frac{n x^{4n-1} \times 2^{m+1} \times (1-\frac{x^4}{2})}{x^{4n}} = \frac{2^{m+1} \times n}{1} = 2^{2n+\frac{1}{2}} \times n = 32\sqrt{2}$$

$\Rightarrow n = 2 \Rightarrow m = 3/5$

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$\Rightarrow 2m + n = 7 + 2 = 9$

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$y = \frac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow \sqrt{x} = \frac{y+1}{y-1} \Rightarrow x = \left(\frac{y+1}{y-1}\right)^2$

$\Rightarrow f^{-1}(x) = \left(\frac{x+1}{x-1}\right)^2 \Rightarrow [f^{-1}(x)]' = 2\left(\frac{x+1}{x-1}\right) \times \left(\frac{-2}{(x-1)^2}\right)$

$\xrightarrow{x=2} [f^{-1}(x)]' = 2 \times 3 \times -2 = -12$

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آهنگ تغییر متوسط $= \frac{f(a) - f(b)}{a-b} = \frac{8(-a+1)-1}{-1-0} = 8a - 7 = -11 \to a = -\frac{1}{2}$

$f(x) = (x^2+1)^3(-\frac{1}{2}x + 1) \to f'(x) = 3(x^2+1)^2 \times 2x(-\frac{1}{2}x+1) + (-\frac{1}{2})(x^2+1)^3$

$= 3(2)^2 \times 2(\frac{1}{2}) + (-\frac{1}{2})(8)$

$a = -2a = 1$

$= 12 + (-4) = 8$

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آهنگ متوسط تغییر $= \dfrac{f(a)-f(b)}{a-b} \Rightarrow y=\sin x\cos^3 x \quad [\frac{\pi}{4}, \frac{\pi}{2}] \rightarrow$ آهنگ متوسط $= \dfrac{\sin\frac{\pi}{4}\cos\frac{\pi}{2}-\sin\frac{\pi}{2}\cos\ldots}{\frac{\pi}{4}-\frac{\pi}{2}}$

$$= \frac{0-(-1)}{-\frac{\pi}{4}} = -\frac{4}{\pi} \quad \mathrm{I}$$

$$y=\sin^4 x-\cos^4 x=\overbrace{(\sin^2 x-\cos^2 x)}^{-\cos 2x}\overbrace{(\cos^2 x+\sin^2 x)}^{1}=-\cos 2x$$

آهنگ متوسط تغییر $= \dfrac{-\cos\frac{\pi}{2}-(-\cos\pi)}{\frac{\pi}{4}-\frac{\pi}{2}} = \dfrac{0-(-(-1))}{-\frac{\pi}{4}} = \dfrac{-1}{-\frac{\pi}{4}} = \dfrac{4}{\pi} \quad \mathrm{II}$

$$\mathrm{I},\mathrm{II} \Rightarrow \frac{-\frac{4}{\pi}}{\frac{4}{\pi}} = \boxed{-1}$$