به نام خدا

تکلیف شماره ۲۳

کیوان سکارم شب

(1)

$g(x)=f(\tan^2x+\sqrt2\cos x) \longrightarrow g'(x)=(\tan^2x+\sqrt2\cos x)'\,f'(\tan^2x+\sqrt2\cos x)$

$=\left(2(\tan^2x+1)(\tan x)\times\sqrt2\cos x+(-\sqrt2\sin x)(\tan^2x)\right)\times f'(\tan^2x+\sqrt2\cos x)$

$\Rightarrow g'(\frac{\pi}{4})=\left(2(1+1)(1)\times\sqrt2(\frac{1}{\sqrt2})+(-\sqrt2\times\frac{1}{\sqrt2})(1)\right)\times f'(1+\sqrt2\times\frac{1}{\sqrt2})$

$\Rightarrow g'(\frac{\pi}{4})=(\underbrace{4+(-1)}_{3})\times f'(2) \longrightarrow \sqrt3=3\times f'(2) \longrightarrow \boxed{f'(2)=\frac{\sqrt3}{3}}$

(2)

$f(x)-2g(x)=\frac{8+\cos^3x}{4-\cos^2x}-2\,\frac{2}{2-\cos x}=\frac{(\cancel{2+\cos x})(4+\cos^2x-2\cos x)}{(2-\cos x)(\cancel{2+\cos x})}-\frac{4}{2-\cos x}$

$=\frac{\cos^2x-2\cos x+4}{2-\cos x}-\frac{4}{2-\cos x}=\frac{\cos^2x-2\cos x}{2-\cos x}=\frac{\cos x(\cos x-2)}{2-\cos x}$

$=-\cos x \longrightarrow (f-2g)'(x)=-(-\sin x)=\sin x$

$\Rightarrow (f-2g)'(\frac{7\pi}{6})=\sin(7\frac{\pi}{6})=-\frac{1}{2}$ ✓

(3)

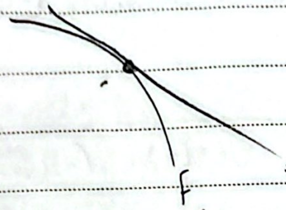
① $\lim\limits_{x\to\frac{3}{4}^+}\frac{(4x-3)\sqrt{ax}}{x-\frac{3}{4}}=\lim\limits_{x\to\frac{3}{4}^+}\frac{4(\cancel{4x-3})\sqrt{ax}}{(\cancel{4x-3})}=\frac{4\sqrt{3a}}{2}=2\sqrt{3a}$

② $\lim\limits_{x\to\frac{3}{4}^-}\frac{-(4x-3)\sqrt{ax}}{x-\frac{3}{4}}=-2\sqrt{3a} \longrightarrow 2\sqrt{3a}-(-2\sqrt{3a})=2\sqrt6$

$\longrightarrow 4\sqrt{3a}=2\sqrt6 \longrightarrow 2\sqrt{3a}=\sqrt6 \longrightarrow \sqrt{3a}=\frac{\sqrt6}{2}$

$\longrightarrow 3a=\frac{6}{4} \longrightarrow \boxed{a=\frac{1}{2}}$ ✓

(٤)



$\to$ ① $f'(4)=y'(4)$ و ② $f(4)=y(4)$

$\to$ ① $f'(4)=-a$

② $f(4)=-4a+2$

$\to f(4)+f'(4)=-1 \to -a-4a+2=-1 \to -5a+2=-1$

$\to +5a=3 \to a=\frac{3}{5} \to f'(4)=-a=-\frac{3}{5}$ ✓

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(٥)

$f(x)=\frac{ax-1}{3x+1} \to f'(x)=\frac{a+3}{(3x+1)^2}$ ①

①=② $\frac{a+3}{(3x+1)^2}=\frac{1}{7}$

$y=\frac{1}{7}x+\frac{5}{7} \to y'=\frac{1}{7}$ ②

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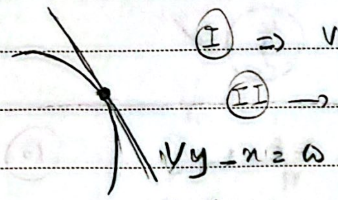

I $\Rightarrow 7a+21=9x^2+6x+1 \to 9x^2+6x-7a-20=0$

II $\to \frac{ax-1}{3x+1}=\frac{x+5}{7} \to 7ax-7=3x^2+x+15x+5$

$\to 3x^2+(16-7a)x+12=0 \to 16-7a=2 \to 14=7a \to \boxed{a=2}$

I $\xrightarrow{\div 3} 3x^2+2x-\left(\frac{7a+20}{3}\right)=0$ ✓

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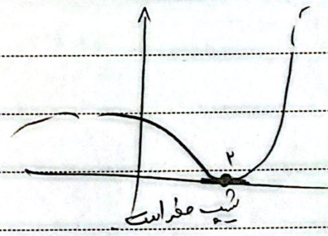
(٦)



طبق شکل $\to f(2)=0$, $f'(2)=0$

① $8+2a-b=0 \to 2a-b=-8$

② $\to f'(x)=3x^2+a \to 0=12+a \to \boxed{a=-12}$

$\to -24-b=-8 \to \boxed{b=-16} \to b-a=-16+12=-4$ ✓

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(٧)

طبق هم‌ارزی داریم ؟ $f'(x)=n(x^2)^{n-1}\times 2x\times\cos x^2$ و $f'(x)=n\sin^{n-1}(x^2)\times 2x\cos x^2$

$x\to0$

$\to 1-\cos x \sim \frac{x^2}{2} \to (1-\cos x)^m=\left(\frac{x^2}{2}\right)^m \to \lim_{x\to0}\frac{a^{2n}\times x^{2n-2}\times n\times 2x\times\cos x^2}{\frac{x^{2m}}{2^m}}$

$=+32\sqrt{2} \to \lim_{x\to0}\frac{x^{4n-1}\times 2n\times(\cos x^2)\times 2^m}{x^{2m}}=32\sqrt{2}=2^5\times 2^{\frac{1}{2}}=2^{\frac{11}{2}}$

$=\lim_{x\to0}\frac{x^{4n-1}\times n\times 2^{m+1}}{x^{2m}}=2^{\frac{11}{2}} \to$ ① $4n-1=2m$ ② $n\times 2^{m+1}=2^{\frac{11}{2}} \to n=2^{\frac{11}{2}-m-1}$

Year...... Month...... Day...... Subject:......

(۸) جدول، واقعا (عادی): $x=9$ $\Leftarrow$ $\frac{\sqrt{x}+1}{\sqrt{x}-1}=۲$ $\Leftarrow$ $f(a)=۲$ $\Leftarrow$ $(f^{-1})'(۲)=\frac{1}{f'(a)}$

$(f^{-1})'(۲)=\frac{1}{f'(9)}$ $\Leftarrow$ $f'(x)=\frac{\frac{\sqrt{x}-1}{۲\sqrt{x}}-\frac{\sqrt{x}+1}{۲\sqrt{x}}}{(\sqrt{x}-1)^۲}=\frac{-۲}{(\sqrt{x}-1)^۲(۲\sqrt{x})}=\frac{-1}{(\sqrt{x})(\sqrt{x}-1)^۲}$

$\xrightarrow{x=9} f'(9)=\frac{-1}{۳\times(\cancel{۳}-1)^۲}=\frac{-1}{۱۲} \Rightarrow (f^{-1})'(۲)=\frac{1}{\frac{-1}{۱۲}}=-۱۲$ ✓✓

(۹) $\frac{f(0)-f(-1)}{+1}=-۱۱ \Rightarrow \frac{1-(۸)(-a+1)}{1}=-۱۱ \Rightarrow \frac{1+۸a-۸}{1}=-۱۱$

$\Rightarrow -۱۱=۸a-۷ \Rightarrow ۸a=-۴ \Rightarrow a=-\frac{1}{۲}$ ✓ $\Rightarrow -۲a=1$

$\Rightarrow f'(x)=۳(x^۲+1)(۲x)\times(-\frac{x}{۲}+1)+(x^۲+1)^۳(-\frac{1}{۲}) \Rightarrow f'(1)=۳(۲)(۲)\times(-\frac{1}{۲}+1)$

$+(۲)^۳(-\frac{1}{۲})=(۱۲\times\frac{1}{۲})+(۸\times-\frac{1}{۲})=۶-۴=۲$ ✓

(۱۰) (۱) $\frac{f(\frac{\pi}{۲})-f(\frac{\pi}{۴})}{\frac{\pi}{۲}-\frac{\pi}{۴}}=\frac{(\sin\frac{\pi}{۲}\cos\pi)-(\cancel{\sin\frac{\pi}{۴}\cos\frac{\pi}{۲}})}{\frac{\pi}{۴}}=\frac{-۴}{\pi}$

می‌دانیم $\sin^۴x-\cos^۴x=-(\cos^۲x-\sin^۲x)(\cancel{\cos^۲x+\sin^۲x})=-\cos ۲x$

$\Rightarrow$ (۲) $\frac{-\cos(\pi)-(\cancel{-\cos\frac{\pi}{۲}})}{\frac{\pi}{۲}-\frac{\pi}{۴}}=\frac{1}{\frac{\pi}{۴}}=\frac{۴}{\pi}$

$\Rightarrow \frac{(۱)}{(۲)}=\frac{\frac{-۴}{\pi}}{\frac{۴}{\pi}}=-1$ ✓