به نام خدا

تکلیف شماره ۲۳ — کیوان سکارم شب — ۱۸٫۵

① $g(x) = f(\tan^2 x + \sqrt{2}\cos x) \Rightarrow g'(x) = (\tan^2 x + \sqrt{2}\cos x)'\, f'(\tan^2 x + \sqrt{2}\cos x)$

$= \left(2(\tan^2 x + 1)(\tan x) \times \sqrt{2}\cos x + (-\sqrt{2}\sin x)(\tan^2 x)\right) \times f'(\tan^2 x + \sqrt{2}\cos x)$

$\Rightarrow g'(\frac{\pi}{4}) = \left(2(1+1)(1) \times \sqrt{2}(\frac{1}{\sqrt{2}}) + (-\sqrt{2} \times \frac{1}{\sqrt{2}})(1)\right) \times f'(1 + \sqrt{2} \times \frac{1}{\sqrt{2}})$ ۲

$\Rightarrow g'(\frac{\pi}{4}) = (\underbrace{\cancel{4 + (-1)}}_{3}) \times f'(2) \Rightarrow \sqrt{3} = 3 \times f'(2) \Rightarrow \boxed{f'(2) = \frac{\sqrt{3}}{3}}$

② $f(x) - 2g(x) = \frac{8 + \cos^3 x}{4 - \cos^2 x} - 2\frac{2}{2 - \cos x} = \frac{(\cancel{2+\cos x})(4 + \cos^2 x - 2\cos x)}{(2 - \cos x)(\cancel{2 + \cos x})} - \frac{4}{2 - \cos x}$

$= \frac{\cos^2 x - 2\cos x + 4}{2 - \cos x} - \frac{4}{2 - \cos x} = \frac{\cos^2 x - 2\cos x}{2 - \cos x} = \frac{\cos x(\cos x - 2)}{2 - \cos x}$

$= -\cos x \Rightarrow (f - 2g)'(x) = -(-\sin x) = \sin x$

$\Rightarrow (f - 2g)'(\frac{7\pi}{6}) = \sin(7\frac{\pi}{6}) = -\frac{1}{2}$ ۲

③

① $\lim_{x \to \frac{3}{4}^+} \frac{(4x-3)\sqrt{ax}}{x - \frac{3}{4}} = \lim_{x \to \frac{3}{4}^+} \frac{4\cancel{(4x-3)}\sqrt{ax}}{\cancel{(4x-3)}} = \frac{4\sqrt{3a}}{2} = 2\sqrt{3a}$

② $\lim_{x \to \frac{3}{4}^-} \frac{-(4x-3)\sqrt{ax}}{x - \frac{3}{4}} = -2\sqrt{3a} \Rightarrow 2\sqrt{3a} - (-2\sqrt{3a}) = 2\sqrt{6}$

$\Rightarrow 4\sqrt{3a} = 2\sqrt{6} \Rightarrow 2\sqrt{3a} = \sqrt{6} \Rightarrow \sqrt{3a} = \frac{\sqrt{6}}{2}$

$\Rightarrow 3a = \frac{6}{4} \Rightarrow \boxed{a = \frac{1}{2}}$ ۲

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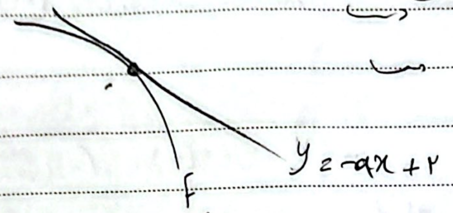

→ ① $f'(4)=y'(4)$ و ② $f(4)=y(4)$

→ ① $f'(4)=-a$

② $f(4)=-4a+2$

$\to f(4)+f'(4)=-1 \to -a-4a+2=-1 \to -5a+2=-1$ ۲

$\to +5a=3 \to a=\frac{3}{5} \to f'(4)=-a=-\frac{3}{5}$ ✓

۵

$f(x)=\frac{ax-1}{3x+1} \to f'(x)=\frac{a+3}{(3x+1)^2}$ ①

①=② $\frac{a+3}{(3x+1)^2}=\frac{1}{7}=$

$y=\frac{1}{7}x+\frac{5}{7} \to y'=\frac{1}{7}$ ②

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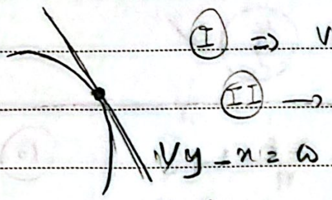

(I) $\Rightarrow 7a+21=9x^2+6x+1 \to 9x^2+6x-7a-20=0$

(II) $\to \frac{ax-1}{3x+1}=\frac{x+5}{7} \to 7ax-7=3x^2+x+15x+5$

$\to 3x^2+(16-7a)x+12=0 \to 16-7a=2 \to 14=7a \to \boxed{a=2}$

(I) $\xrightarrow{\div 3} 3x^2+2x-\left(\frac{7a+20}{3}\right)=0$

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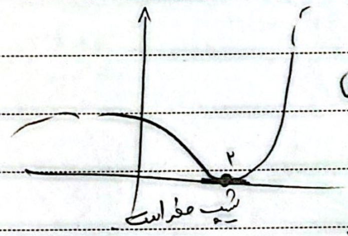

طبق شکل → $f(2)=0$, $f'(2)=0$

① $8+2a-b=0 \to 2a-b=-8$ ۲

② $\to f'(x)=3x^2+a \to 0=12+a \to \boxed{a=-12}$

$\to -24-b=-8 \to \boxed{b=-16} \to b-a=-16+12=-4$ ✓

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طبق همارزی داریم؟ $f'(x)=n\sin^{n-1}(x^2)\times 2x\cos x^2$, $f'(x)=n(x^2)^{n-1}\times 2x\times\cos x^2$
$x\to 0$

$\xrightarrow{x\to 0} 1-\cos x \sim \frac{x^2}{2} \to (1-\cos x)^m=\left(\frac{x^2}{2}\right)^m \to \lim_{x\to 0}\frac{x^{2n}\times x^{2n-2}\times n\times 2x\times\cos x^2}{\frac{x^{2m}}{2^m}}$

۱,۵

$=+32\sqrt{2} \to \lim_{x\to 0}\frac{x^{4n-1}\times 2n\times(\cos x^2)\times 2^m}{x^{2m}}=32\sqrt{2}=2^5\times 2^{\frac{1}{2}}=2^{\frac{11}{2}}$

$=\lim_{x\to 0}\frac{x^{4n-1}\times n\times 2^{m+1}}{x^{2m}}=2^{\frac{11}{2}} \to$ ① $4n-1=2m$

② $n\times 2^{m+1}=2^{\frac{11}{2}} \to n=2^{\frac{11}{2}-m-1}$

ادامه؟

Year .... Month .... Day .... Subject: ....

۸) حدس وارون بیلی؛ $x = 9$

$(f^{-1})'(2) = \dfrac{1}{f'(a)} \rightarrow f(a) = 2 \rightarrow \dfrac{\sqrt{x}+1}{\sqrt{x}-1} = 2 \rightarrow x = 9$

$(f^{-1})'(2) = \dfrac{1}{f'(9)} \rightarrow f'(x) = \dfrac{\dfrac{\sqrt{x}-1}{2\sqrt{x}} - \dfrac{\sqrt{x}+1}{2\sqrt{x}}}{(\sqrt{x}-1)^2} = \dfrac{-2}{(\sqrt{x}-1)^2(2\sqrt{x})} = \dfrac{-1}{(\sqrt{x})(\sqrt{x}-1)^2}$ ۲

$\xrightarrow{x=9} f'(9) = \dfrac{-1}{3 \times (3-1)^2} = -\dfrac{1}{12} \rightarrow (f^{-1})'(2) = \dfrac{1}{-\frac{1}{12}} = -12$ ✓✓

۹) $\dfrac{f(0) - f(-1)}{+1} = -11 \rightarrow \dfrac{1 - (8)(-a+1)}{1} = -11 \rightarrow \dfrac{1 + 8a - 8}{1} = -11$

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$\rightarrow -11 = 8a - 7 \rightarrow 8a = -4 \rightarrow a = -\dfrac{1}{2}$ ✓ $\rightarrow -2a = 1$

$\rightarrow f'(x) = 3(x^2+1)^2(2x) \times (-\dfrac{x}{2}+1) + (x^2+1)^3(-\dfrac{1}{2}) \Rightarrow f'(1) = 3(2)(2) \times (-\dfrac{1}{2}+1)$

$+ (2)^3(-\dfrac{1}{2}) = (12 \times +\dfrac{1}{2}) + (8 \times -\dfrac{1}{2}) = 6 - 4 = 2$ ✓

۱۰) ۱) $\dfrac{f(\frac{\pi}{2}) - f(\frac{\pi}{4})}{\frac{\pi}{2} - \frac{\pi}{4}} = \dfrac{(\sin\frac{\pi}{2}\cos\pi) - \cancel{(\sin\frac{\pi}{4}\cos\frac{\pi}{2})}^{\,0}}{\frac{\pi}{4}} = \dfrac{-4}{\pi}$

می دانیم $\sin^4 x - \cos^4 x = -(\cos^2 x - \sin^2 x)\cancel{(\cos^2 x + \sin^2 x)}^{\,1} = -\cos 2x$

$\Rightarrow$ ۲) $\dfrac{-\cos(\pi) - \cancel{(-\cos\frac{\pi}{2})}^{\,0}}{\frac{\pi}{2} - \frac{\pi}{4}} = \dfrac{1}{\frac{\pi}{4}} = \dfrac{4}{\pi}$

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$\rightarrow \dfrac{1}{2} = \dfrac{-\frac{4}{\pi}}{\frac{4}{\pi}} = -1$ ✓

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$$\frac{ax-1}{3x+1} = \frac{x}{7}+\frac{5}{7} \longrightarrow 7ax-7 = 3x^2+x+15x+5$$

$$3x^2 + x(16-7a)+12=0 \xrightarrow{\Delta=0} (16-7a)^2-12^2 = (16-7a-12)(16-7a+12)$$

$a=\frac{4}{7}$, $a=4$

$\boxed{a=4} \longrightarrow 3x^2-12x+12 \longrightarrow \boxed{x=2}$ ✓

$a=\frac{4}{7} \longrightarrow 3x^2+12x+12 \longrightarrow x=-2$ ناحیه اول ✗

۷

$$\lim_{x\to 0} \frac{\sin^n(x^2) \times 2x\sin^{n-1}(x^2)\times \cos x^2}{(2\sin^2\frac{x}{2})^m} \xrightarrow{\text{هم ارزی}} \lim_{x\to 0}\frac{(x^2)^n \times n \times (x^2)^{n-1}\times 2x}{(2(\frac{x}{2})^2)^m} = \frac{(x)^{2n+2n-2+1}\times 2n}{(x)^{2m}\times\frac{1}{2}}$$

$$= \frac{2nx^{4n-1}}{\frac{1}{2^m}\times x^{2m}} = 2^{m+1}\times n\times x^{4n-2m-1}$$

$\Rightarrow 4n-2m-1=0 \longrightarrow n=\frac{2m+1}{4}$

$\Rightarrow 2^{m+1}\times\frac{2m+1}{4} = 32\sqrt{2} \longrightarrow m=\frac{7}{2}\ ,\ n=2 \longrightarrow \boxed{2m+n=9}$

۹

$$\frac{f(0)-f(-1)}{0-(-1)} = \frac{1-8(-a+1)}{1} = 8a-7=-11 \longrightarrow 8a=-4 \longrightarrow \boxed{a=-\frac{1}{2}}$$

$$f'(x) = 3(x^2+1)^2(2x)(ax+1) + a(x^2+1)^3 \xrightarrow{-2a=1} f'(1) = 3(2)^2(2)(\frac{1}{2}) + -\frac{1}{2}(2)^3$$

$$f'(1) = 12-4=\boxed{8}$$