نام و نام خانوادگی .... پاسخنامه تشریحی تکلیف شماره .... کلاس ....

نام و نام خانوادگی نوشته نشده است نمره ای لحاظ نمی‌شود

صحیح

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$g(x) = f(u) \Rightarrow g'(x) = f'(u)\,u'$

$\Rightarrow f'(\tan^2 x + \sqrt{2}\cos x)(2\tan x(1-\tan^2 x) - \sqrt{2}\sin x) = f'(2) = \sqrt{3}$

$\downarrow 1 \qquad \downarrow 0 \qquad -1$

$f'(2) = -\sqrt{3}$

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$$f'(x) = \frac{-3\cos^2 x \sin x(4-\cos^2 x) + (8+\cos^3 x)(2\sin x\cos x)}{(4-\cos^2 x)^2} \Rightarrow f'(\frac{\pi}{3}) = \frac{}{\frac{15}{4}}$$

$-3 \times \frac{3}{4} \times \frac{1}{2} \times \frac{15}{4} + (\frac{3\sqrt{3}}{8} + 8) \times \frac{1}{4}$

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$x = \frac{3}{4} \xrightarrow{\text{نقطه گوشه}} x > \frac{3}{4} \quad (4x-3)\sqrt{ax} \rightarrow f' = 4\sqrt{ax} + (4x-3)\frac{a}{2\sqrt{ax}} =$

$f' = 2\sqrt{3a}$

$x < \frac{3}{4} \quad (-4x+3)\sqrt{ax} \longrightarrow f' = -4\sqrt{ax}$

$f' = -2\sqrt{3a}$

$|2\sqrt{3a} + 2\sqrt{3a}| = 4\sqrt{3a} = 2\sqrt{6} \Rightarrow \boxed{a = \frac{1}{2}}$

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$y = 2 - ax \Rightarrow \quad f(4) = 2 - 4a \quad f'(4) = -a \quad f'(4) = \frac{-3}{5}$

$f + f'(4) = 2 - 5a = -1 \Rightarrow a = \frac{3}{5}$

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$y' = \frac{a+3}{(3x+1)^2} = \frac{1}{7}$

$\frac{ax-1}{3x+1} = \frac{x+5}{7} \Rightarrow 7ax - 7 = 3x^2 + 16x + 5 \Rightarrow 3x^2 + (16 - 7a)$

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$f(۲)=۰ \rightarrow ۸+۲a-b=۰ \Rightarrow ۲a-b=-۸$

$f'(۲)=۰ \rightarrow ۳\times۴+a=۰ \Rightarrow a=-۱۲ \Rightarrow b=-۱۶$

$-۱۶+۱۲=\boxed{-۴}$

۶ ۲

$\lim \frac{f(x)f'(x)}{(1-\cos x)^m}=۳۲\sqrt{۲} \Rightarrow \frac{n\sin^{n-1}(x^2)\,۲x\sin(x^2)}{(1-\cos x)^m}=۳۲\sqrt{۲}$

۷ ۰

۸ ۰

۹ ۰

۱۰ ۰

$$g'(x) = f'(\tan^2 x + \sqrt{2}\cos x) \times 2\tan x(1+\tan^2 x) - \sqrt{2}\sin x)$$ ۱

$$g'(\tfrac{\pi}{4}) = f'(1 + \sqrt{2}\times\tfrac{\sqrt{2}}{2}) \times 2\times 1 \times (1+1^2) - (\sqrt{2}\times\tfrac{\sqrt{2}}{2})$$

$$\cancel{g'(\tfrac{\pi}{4})}_{\sqrt{3}} = f'(2) \times [\cancel{2(2)-1}_{3}] \longrightarrow f'(2) = \frac{\sqrt{3}}{3}$$

$$f - 2g(\tfrac{7\pi}{4}) = \frac{(2+\cancel{\cos x})(\cos^2 x - 2\cos x + 4)}{(2-\cos x)(2+\cancel{\cos x})} - \frac{4}{2-\cos x} = \frac{\overset{\cos x(\cos x - 2)}{\cos^2 x - \cancel{2\cos x}}}{2-\cancel{\cos x}} = -\cos x$$ ۲

$$(f-2g)' = -(\cos x)' = +\sin x \longrightarrow \sin(\tfrac{7\pi}{4}) = \boxed{-\tfrac{1}{2}}$$

$$x \to \tfrac{3}{4}^+ \leadsto f(x) = 4x - 3\sqrt{\tfrac{3a}{4}} \xrightarrow[\text{صفرشونده}]{\text{مشتق‌گیر}} f'(x) = \frac{4\sqrt{3a}}{2} = 2\sqrt{3a}$$ ۳

$$x \to \tfrac{3}{4}^- \leadsto f(x) = -4x + 3\sqrt{\tfrac{3a}{4}} \xrightarrow[\text{صفرشونده}]{\text{مشتق‌گیر}} f'(x) = \frac{-4\sqrt{3a}}{2} = -2\sqrt{3a}$$

$$4\sqrt{3a} = 2\sqrt{6} \longrightarrow \sqrt{3a} = \frac{\sqrt{6}}{2} \longrightarrow \boxed{a = \tfrac{1}{2}}$$

$$\frac{ax-1}{3x+1} = \frac{x}{7} + \frac{5}{7} \longrightarrow 7ax - 7 = 3x^2 + x + 15x + 5$$ ۵

$$\boxed{a=\tfrac{4}{7}} \qquad \boxed{a=4}$$

$$3x^2 + x(16-7a) + 12 = 0 \xrightarrow{\Delta=0} (16-7a)^2 - 12^2 = (16-7a-12)(16-7a+12)$$

$$\boxed{a=4} \longrightarrow 3x^2 - 12x + 12 \longrightarrow \boxed{x=2}\ \checkmark$$

$$a = \tfrac{4}{7} \longrightarrow 3x^2 + 12x + 12 \longrightarrow x = -2$$ × ناحیه اول

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$$\lim_{x\to 0}\frac{\sin^n(x^2)\times 2x\sin^{n-1}(x^2)\times\cos x^2}{(2\sin^2\frac{x}{2})^m}\xrightarrow{\text{هم ارزی}}\lim_{x\to 0}\frac{(x^2)^n\times x\times(x^2)^{n-1}\times 2x}{(2(\frac{x}{2})^2)^m}=\frac{(x)^{2n+2n-2+1}\times 2x}{(x)^{2m}\times\frac{1}{2}}$$

$$=\frac{2x\,x^{4n-1}}{\frac{1}{2^m}\times x^{2m}}=2^{m+1}\times x\times x^{4n-2m-1}\quad\rightarrow 4n-2m-1=0\rightarrow n=\frac{2m+1}{4}$$

$$\rightarrow 2^{m+1}\times\frac{2m+1}{4}=32\sqrt{2}\longrightarrow m=\frac{7}{2},\ n=2\rightarrow 2m+n=9$$

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$$f^{-1}(2)=t\rightarrow f(t)=2\longrightarrow \sqrt{t}+1=2\sqrt{t}-2\rightarrow\sqrt{t}=3\rightarrow t=9$$

$$(f^{-1})'(2)=\frac{1}{f'(9)}\quad,\quad f'(x)=\frac{-2}{(\sqrt{x}-1)^2}\times\frac{1}{2\sqrt{x}}$$

$$f'(9)=\frac{-2}{(2)^2}\times\frac{1}{6}=-\frac{1}{12}\longrightarrow (f^{-1})'(2)=-12$$

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$$\frac{f(0)-f(-1)}{0-(-1)}=\frac{1-8(-a+1)}{1}=8a-7=-11\rightarrow 8a=-4\rightarrow\boxed{a=-\frac{1}{2}}$$

$$f'(x)=3(x^2+1)^2(2x)(ax+1)+a(x^2+1)^3\xrightarrow{-2a=1}f'(1)=3(2)^2(2)(\tfrac{1}{2})+-\tfrac{1}{2}(2)^3$$

$$f'(1)=12-4=8$$

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$$x=\frac{\pi}{4}\rightarrow\sin\frac{\pi}{4}\cos\frac{\pi}{2}=0$$
$$x=\frac{\pi}{2}\rightarrow\sin\frac{\pi}{2}\cos\pi=-1\quad\rightarrow\frac{-1-0}{\frac{\pi}{2}}=\boxed{-\frac{4}{\pi}=\frac{\Delta y}{\Delta x}}$$

$$\sin^4x-\cos^4x=(\sin^2x\overset{1}{\cancel{+}\cos^2x})(\underbrace{\sin^2x-\cos^2x}_{-\cos 2x})=-\cos 2x$$

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$$x=\frac{\pi}{4}\rightarrow-\cos\frac{\pi}{2}=0$$
$$x=\frac{\pi}{2}\rightarrow-\cos\pi=1\quad\rightarrow\frac{1-0}{\frac{\pi}{4}}=\boxed{\frac{4}{\pi}=\frac{\Delta y}{\Delta x}}\longrightarrow\frac{-\frac{4}{\pi}}{\frac{4}{\pi}}=-1$$