نام و نام خانوادگی ....[illegible]........ پاسخنامه تشریحی تکلیف شماره ...۲۳... کلاس ...دوازدهم [illegible] B........

۱

$$g(x)=f\left(\tan^2 x+\sqrt{2}\cos x\right)$$

$$g'(x)=\left(2\tan x(1+\tan^2 x)-\sqrt{2}\sin x\right)\times f'\left(\tan^2 x+\sqrt{2}\cos x\right)$$

$$g'\left(\frac{\pi}{4}\right)=\left(2\times 1\times 2-\frac{\sqrt{2}\times\sqrt{2}}{2\times 1}\right)\times f'\left(1+\frac{\sqrt{2}\times\sqrt{2}}{2}\right)=3f'(2)=\sqrt{3}$$

$$f'(2)=\frac{\sqrt{3}}{3}$$

۲

$$f(x)=\frac{8+\cos^3 x}{4-\cos^2 x}\longrightarrow f'(x)=\frac{-3\cos^2 x\sin x(4-\cos^2 x)-(+2\cos x\sin x(8+\cos^3 x))}{(4-\cos^2 x)^2}$$

$$g(x)=\frac{2}{2-\cos x}\longrightarrow g'(x)=\frac{0-\overset{2\sin x}{(\pm\sin x(2))}}{(2-\cos x)^2}$$

$$\underbrace{f'\left(\frac{7\pi}{6}\right)}_{\frac{27-64\sqrt{3}}{169}}-2\underbrace{g'\left(\frac{7\pi}{6}\right)}_{\frac{8}{19+8\sqrt{3}}}=\frac{27-64\sqrt{3}}{169}-\frac{8}{19+8\sqrt{3}}$$

۳

$$f(x)=|4x-3|\sqrt{ax}\longrightarrow f(x)=\begin{cases}4x\sqrt{ax}-3\sqrt{ax} & x\geq\frac{3}{4}\\ 3\sqrt{ax}-4x\sqrt{ax} & x<\frac{3}{4}\end{cases}$$

$$f'(x)=\begin{cases}\frac{12ax-3a}{2\sqrt{ax}} & x\geq\frac{3}{4}\longrightarrow f'_+\left(\frac{3}{4}\right)=\frac{6a}{\sqrt{3a}}\\ \frac{3a-12ax}{2\sqrt{ax}} & x<\frac{3}{4}\longrightarrow f'_-\left(\frac{3}{4}\right)=\frac{-6a}{\sqrt{3a}}\end{cases}$$

$$\frac{6a}{\sqrt{3a}}-\left(\frac{-6a}{\sqrt{3a}}\right)=\frac{12a}{\sqrt{3a}}=2\sqrt{6}\to\sqrt{12a}=\sqrt{6}\to a=\frac{1}{2}$$

۴

$y+ax=2\longrightarrow$ f مماس بر در نقطه ۴ $\longrightarrow$ $f(4)=2-4a$, $f'(4)=-a$

$$\underbrace{f(4)}_{2-4a}+\underbrace{f'(4)}_{-a}=-1\to 2-5a=-1\to 3=5a\to a=\frac{3}{5}\to f'(4)=-a=-\frac{3}{5}$$

۵

$$y=\frac{ax-1}{3x+1}\longrightarrow\frac{ax-1}{3x+1}=\frac{x+5}{7}\quad,\quad\frac{a+3}{(3x+1)^2}=\frac{1}{7}$$

$$7y-x=5\longrightarrow\quad 7ax-7=3x^2+16x+5\quad,\quad 7a+21=9x^2+6x+1$$

$f(x) = x^3 + ax - b \xrightarrow{\text{مماس بر } x \text{ در } 2} f(2) = 8 + 2a - b = 0 \to \underbrace{8 - 24}_{-16} - b = 0 \to b = -16$

$f'(2) = 3x^2 + a = 12 + a = 0 \to a = -12$

$\underset{-16}{b} - \underset{-12}{a} = -4$ 6

$\dfrac{\sqrt{x}+1}{\sqrt{x}-1} = 2 \to \sqrt{x} + 1 = 2\sqrt{x} - 2 \to \sqrt{x} = 3 \to x = 9$

$f'(x) = \dfrac{1}{2\sqrt{x}}(\sqrt{x}-1) - \left(\dfrac{1}{2\sqrt{x}}(\sqrt{x}+1)\right)$

$(f^{-1})'(2) = \dfrac{1}{f'(9)}$ $\quad f'(9) = \dfrac{1}{6}(2) - \dfrac{1}{6}(4) = -\dfrac{1}{3}$ 8

$(f^{-1})'(2) = \dfrac{1}{-\frac{1}{3}} = -3$

جابجا حل کردم

$f(x) = \sin^n(x^2) \to f'(x) = n\sin^{n-1}(x^2)\cos(x^2)\,2x$ $\quad \underset{2}{\cancel{4m}} + \underset{2}{\cancel{n}} = 9$

$\lim\limits_{x\to 0} \dfrac{f(x)f'(x)}{(1-\cos x)^m} = 32\sqrt{2} \to \lim\limits_{x\to 0} \dfrac{\sin^n(x^2)\, n \sin^{n-1}(x^2)\cos(x^2)\,2x}{(\underbrace{1-\cos x}_{2\sin^2\frac{x}{2}})^m} \longrightarrow$ 7

$4n - 1 = 2m \quad m = 2.5$

$\xrightarrow{\text{هم ارزی}} \dfrac{x^{2n}\cdot n \cdot x^{2n-2}\cdot \left(1 - \frac{x^2}{2}\right)\cdot 2x}{\left(1 - 1 + \frac{x^2}{2}\right)^m} = \dfrac{\cancel{x^{4n-1}}\cdot n \cdot 2 \cdot \left(1 - \frac{x^2}{2}\right)}{\cancel{\left(\frac{x^2}{2}\right)^m}} = 2n\, 2^m = 32\sqrt{2}$

$n 2^{\frac{4n-1}{2}} = 2^{\frac{9}{2}} \quad n = 2$

$f(x) = (x^2+1)^3 (ax+1)$ $\quad \dfrac{\overset{1}{f(0)} - \overset{(8-8a)}{f(-1)}}{1} = -11 \to 8a - 7 = -11 \to a = -\dfrac{1}{2}$

$f'(x) = 3(x^2+1)^2\, 2x\left(-\dfrac{x}{2}+1\right) + -\dfrac{1}{2}(x^2+1)^3 \to f'(\underset{1}{-2a}) = 12 - 4 = 8$ 9

$y = \sin x \cos 2x \longrightarrow \dfrac{\overset{1}{\sin\frac{\pi}{2}}\,\overset{-1}{\cos\pi} - \overset{\frac{\sqrt{2}}{2}}{\sin\frac{\pi}{4}}\,\overset{0}{\cos\frac{\pi}{2}}}{\frac{\pi}{4}} = -\dfrac{4}{\pi}$

$y = \sin^4 x - \cos^4 x \longrightarrow \dfrac{\left(\overset{1}{\sin^4\frac{\pi}{2}} - \overset{0}{\cos^4\frac{\pi}{2}}\right) - \left(\overset{\frac{1}{4}}{\sin^4\frac{\pi}{4}} - \overset{\frac{1}{4}}{\cos^4\frac{\pi}{4}}\right)}{\frac{\pi}{4}} = \dfrac{4}{\pi}$ $\quad$ —— $= -1$ 10