نام و نام خانوادگی ....[illegible].......... پاسخنامه تشریحی تکلیف شماره ..۲۳.. کلاس ...دوازدهم تجربی B........

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$$g(x) = f(\tan^2 x + \sqrt{2}\cos x)$$

$$g'(x) = \left(2\tan x(1+\tan^2 x) - \sqrt{2}\sin x\right) \times f'(\tan^2 x + \sqrt{2}\cos x)$$

$$g'(\tfrac{\pi}{4}) = \left(2\times 1\times 2 - \frac{\sqrt{2}\times\sqrt{2}}{2\times 1}\right) \times f'\left(1 + \frac{\sqrt{2}\times\sqrt{2}}{2}\right) = 3f'(2) = \sqrt{3}$$

$$f'(2) = \frac{\sqrt{3}}{3}$$

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$$f(x) = \frac{8+\cos^3 x}{4-\cos^2 x} \longrightarrow f'(x) = \frac{-3\cos^2 x\sin x(4-\cos^2 x) - (+2\cos x\sin x(8+\cos^3 x))}{(4-\cos^2 x)^2}$$

$$g(x) = \frac{2}{2-\cos x} \longrightarrow g'(x) = \frac{0 - (\pm\sin x(2))}{(2-\cos x)^2}$$

$2\sin x$

$$\underbrace{f'(\tfrac{7\pi}{6})}_{\frac{27-64\sqrt{3}}{169}} - 2\underbrace{g'(\tfrac{7\pi}{6})}_{\frac{8}{19+8\sqrt{3}}} = \frac{27-64\sqrt{3}}{169} - \frac{8}{19+8\sqrt{3}}$$

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$$f(x) = |4x-3|\sqrt{ax} \longrightarrow f'(x) = \begin{cases} 4x\sqrt{ax} - 3\sqrt{ax} & x \geq \frac{3}{4} \\ 3\sqrt{ax} - 4x\sqrt{ax} & x < \frac{3}{4} \end{cases}$$

$$f'(x) = \begin{cases} \frac{12ax-3a}{2\sqrt{ax}} & x \geq \frac{3}{4} \longrightarrow f'_+(\frac{3}{4}) = \frac{6a}{\sqrt{3a}} \\ \frac{3a-12ax}{2\sqrt{ax}} & x < \frac{3}{4} \longrightarrow f'_-(\frac{3}{4}) = \frac{-6a}{\sqrt{3a}} \end{cases}$$

$$\frac{6a}{\sqrt{3a}} - \left(\frac{-6a}{\sqrt{3a}}\right) = \frac{12a}{\sqrt{3a}} = 2\sqrt{6} \to \sqrt{12a} = \sqrt{6} \to a = \frac{1}{2}$$

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$y + ax = 2 \longrightarrow$ f مماس بر در نقطه ۴ $\longrightarrow f(4) = 2-4a$, $f'(4) = -a$

$$\underbrace{f(4)}_{2-4a} + \underbrace{f'(4)}_{-a} = -1 \to 2-5a = -1 \to 3 = 5a \to a = \frac{3}{5} \to f'(4) = -a = -\frac{3}{5}$$

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$$y = \frac{ax-1}{3x+1} \longrightarrow \frac{ax-1}{3x+1} = \frac{x+5}{7} \quad , \quad \frac{a+3}{(3x+1)^2} = \frac{1}{7}$$

$$7y - x = 5 \longrightarrow$$

$$7ax - 7 = 3x^2 + 16x + 5$$

$$7a + 21 = 9x^2 + 6x + 1$$

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$f(x) = x^3 + ax - b$ $\xrightarrow{\text{مماس بر } x \text{ در } 2}$ $f(2) = 8 + 2a - b = 0 \rightarrow \underbrace{8 - 24}_{-16} - b = 0 \rightarrow b = -16$

$f'(2) = 3x^2 + a = 12 + a = 0 \rightarrow a = -12$ ۶

$\underset{-16}{b} - \underset{-12}{a} = -4$

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$\dfrac{\sqrt{x}+1}{\sqrt{x}-1} = 2 \rightarrow \sqrt{x}+1 = 2\sqrt{x} - 2 \rightarrow \sqrt{x} = 3 \rightarrow x = 9$

$f'(x) = \frac{1}{2\sqrt{x}}(\sqrt{x}-1) - \left(\frac{1}{2\sqrt{x}}(\sqrt{x}+1)\right)$

$(f^{-1})'(2) = \dfrac{1}{f'(9)}$ $\quad f'(9) = \frac{1}{6}(2) - \frac{1}{6}(4) = -\frac{1}{3}$ ۸

$(f^{-1})'(2) = \dfrac{1}{-\frac{1}{3}} = -3$

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جا به جا حل کردم

$f(x) = \sin^n(x^2) \rightarrow f'(x) = n\sin^{n-1}(x^2)\cos(x^2)2x$ $\quad \underset{2}{4m} + \underset{2}{n} = 9$

$\lim\limits_{x\to 0} \dfrac{f(x)f'(x)}{(1-\cos x)^m} = 32\sqrt{2} \rightarrow \lim\limits_{x\to 0} \dfrac{\sin^n(x^2)\, n\sin^{n-1}(x^2)\cos(x^2)2x}{(\underbrace{1-\cos x}_{2\sin^2\frac{x}{2}})^m} \longrightarrow$ ۷

$\rightarrow 4n - 1 = 2m \quad m = 3/5$

$\xrightarrow{\text{هم ارزی}} \dfrac{x^{2n}\cdot n\cdot x^{2n-2}\cdot \left(1 - \frac{x^2}{2}\right)\cdot 2x}{\left(1 - 1 + \frac{x^2}{4}\right)^m} = \dfrac{\cancel{x^{4n-1}}\cdot n\cdot 2\cdot\left(1-\frac{x^2}{2}\right)}{\cancel{\left(\frac{x^2}{2}\right)^m}} = 4n\, 2^m = 32\sqrt{2}$

$n 2^{\frac{4n-1}{2}} = 2^{\frac{9}{2}}$ $\quad n = 2$

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$f(x) = (x^2+1)^3(ax+1)$ $\quad \dfrac{\overset{1}{f(0)} - \overset{(8-8a)}{f(-1)}}{1} = -11 \rightarrow 8a - 7 = -11 \rightarrow a = -\frac{1}{2}$

$f'(x) = 3(x^2+1)^2 2x\left(-\frac{x}{2}+1\right) + -\frac{1}{2}(x^2+1)^3 \rightarrow f'(1) = 12 - 4 = 8$ ۹

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$y = \sin x \cos 2x \longrightarrow \dfrac{\overset{-1}{\sin\frac{\pi}{2}\cos\pi} - \overset{0}{\sin\frac{\pi}{4}\cos\frac{\pi}{2}}}{\frac{\pi}{4}} = -\frac{4}{\pi}$

$y = \sin^4 x - \cos^4 x \longrightarrow \dfrac{\left(\sin^4\frac{\pi}{2} - \cos^4\frac{\pi}{2}\right) - \left(\sin^4\frac{\pi}{4} - \cos^4\frac{\pi}{4}\right)}{\frac{\pi}{4}} = \frac{4}{\pi}$ $\quad = -1$ ۱۰

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$$f-۲g(\frac{۷\pi}{۴}) = \frac{(۲+\cancel{\cos x})(\cos^۲ x-۲\cos x+۴)}{(۲-\cos x)(۲+\cancel{\cos x})} - \frac{۴}{۲-\cos x} = \frac{\overset{\cos x(\cancel{\cos x}-۲)}{\cos^۲ x-\cancel{۲\cos x}}}{۲-\cancel{\cos x}} = -\cos x$$ ۲

$$(f-۲g)' = -(\cos x)' = +\sin x \longrightarrow \sin(\frac{۷\pi}{۴}) = \boxed{-\frac{۱}{۲}}$$

$$\frac{ax-۱}{۳x+۱} = \frac{x}{۷}+\frac{۵}{۷} \longrightarrow ۷ax-۷ = ۳x^۲+x+۱۵x+۵$$ ۵

$$۳x^۲+x(۱۶-۷a)+۱۲=۰ \xrightarrow{\Delta=۰} (۱۶-۷a)^۲-۱۲^۲ = \underbrace{(۱۶-۷a-۱۲)}_{\boxed{a=\frac{۴}{۷}}}\underbrace{(۱۶-۷a+۱۲)}_{\boxed{a=۴}}$$

$\boxed{a=۴} \longrightarrow ۳x^۲-۱۲x+۱۲ \longrightarrow \boxed{x=۲}$ ✓

$a=\frac{۴}{۷} \longrightarrow ۳x^۲+۱۲x+۱۲ \longrightarrow x=-۲$ ناحیه اول ✗

$$f^{-۱}(۲)=t \to f(t)=۲ \longrightarrow \sqrt{t}+۱ = ۲\sqrt{t}-۲ \to \sqrt{t}=۳ \to t=۹$$ ۸

$$f^{-۱}(۲)' = \frac{۱}{f'(۹)} \quad , \quad f'(x) = \frac{-۲}{(\sqrt{x}-۱)^۲}\times\frac{۱}{۲\sqrt{x}}$$

$$f'(۹) = \frac{-۲}{(۲)^۲}\times\frac{۱}{۶} = -\frac{۱}{۱۲} \longrightarrow \boxed{f^{-۱}(۲)' = -۱۲}$$