نام و نام خانوادگی: هومن ... پاسخنامه تشریحی تکلیف شماره ۲۳ کلاس ... B

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$g(x)=f(\tan^2x+\sqrt{2}\cos x) \Rightarrow g'(x)=2(\tan x)(1+\tan^2x)-\sqrt{2}\sin x\times f'(\tan^2x+\sqrt{2}\cos x)$

$\Rightarrow g'(\frac{\pi}{4})=2(1)(2)-\sqrt{2}(\frac{\sqrt{2}}{2})\times f'(2)=\sqrt{3}$

$\Rightarrow (4-1)f'(2)=\sqrt{3} \Rightarrow f'(2)=\frac{\sqrt{3}}{3}$

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$f(x)=\frac{(2+\cos x)(4+\cos^2x-2\cos x)}{(2-\cos x)(2+\cos x)} \Rightarrow f(x)=\frac{4+\cos^2x-2\cos x}{2-\cos x}$

$\xrightarrow{-2g(x)} \frac{-2\cos x+\cos^2x}{2-\cos x}=-\cos x \xrightarrow{(f-2g)'} -(\sin x)=(f-2g)'(x)$

$\Rightarrow (f-2g)'(\frac{7\pi}{6})=\sin\frac{7\pi}{6}=-\frac{1}{2}$

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$f_-(\frac{3}{4})=(-4x+3)\sqrt{ax} \xrightarrow{f'(x)} -4\sqrt{ax}=-2\sqrt{3a}$

$f_+(\frac{3}{4})=(4x-3)\sqrt{ax} \xrightarrow{f'(x)} 4\sqrt{ax}=4\sqrt{\frac{3a}{4}}=2\sqrt{3a}$

$\xrightarrow{\text{اختلاف}} 2\sqrt{6}=4\sqrt{3a} \Rightarrow \sqrt{6}=2\sqrt{3a} \Rightarrow 12a=6 \Rightarrow a=\frac{1}{2}$

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$d(x)=-ax+2$

$f'(4)=-a \rightarrow f(4)+f'(4)=-1$

$f(4)=-4a+2$

$\Rightarrow -4a+2-a=-1$

$\Rightarrow 5a=3$

$\rightarrow f'(4)=-a=-\frac{3}{5} \qquad \Rightarrow a=\frac{3}{5}$

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$7y=x+5$

$7y=\frac{7ax-7}{3x+1}$

$\Rightarrow 3x^2+16x+5=7ax-7 \Rightarrow 3x^2+(16-7a)x+12=0$

$\xrightarrow[\text{شرط مماس بودن}]{\Delta=0} (16-7a)^2-4(3)(12)=0 \Rightarrow (16-7a)=12 \Rightarrow a=\frac{4}{7}$

$(16-7a)=-12 \Rightarrow a=4$

$\Rightarrow a\begin{cases}\frac{4}{7} \text{ غ ق ق} \rightarrow \text{ریشه منفی} & -\frac{b}{a}<0\\ 4 \text{ ق ق} \rightarrow \text{ریشه مثبت} & -\frac{b}{a}>0\end{cases}$

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$f'(x) = 3x^2 + a \Rightarrow f'(2) = 12 + a = 0 \Rightarrow a = -12$

$f(x) = x^3 - 12x - b \Rightarrow f(2) = 0 \Rightarrow 8 - 24 - b = 0 \Rightarrow b = -16$

$b - a = -16 + 12 = -4$ ✓

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$f'(x) = \dfrac{\frac{1}{2\sqrt{x}}(\sqrt{x} - 1 - \sqrt{x} - 1)}{(\sqrt{x} - 1)^2} = \dfrac{\frac{-1}{\sqrt{x}}}{(\sqrt{x}-1)^2}$

$f^{-1}(2) = x \Rightarrow f(x) = 2 \Rightarrow x = 9$

$\xrightarrow{x=9} \dfrac{\frac{-1}{\sqrt{9}}}{(\sqrt{9}-1)^2} = \dfrac{-1}{3(3-1)^2} = \dfrac{-1}{3 \times 4} = \dfrac{-1}{12} \Rightarrow (f^{-1}(x))' = \dfrac{1}{m} = -12$ ✓

$f'(2) = 9$

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$f(0) = 1 \quad f(-1) = 8(1-a) = 8 - 8a \Rightarrow \dfrac{1 - (8)(1-a)}{0 - (-1)} = \dfrac{8a - 7}{1} = -11$

$\Rightarrow 8a - 7 = -11 \Rightarrow 8a = -4 \Rightarrow a = -\dfrac{1}{2}$ ✓ $\Rightarrow x = -2a = -2\left(-\dfrac{1}{2}\right) = 1$

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$f'(x) = 3(x^2+1)^2(2x)\left(1 - \dfrac{1}{2}x\right) + \left(\dfrac{-1}{2}\right)(x^2+1)^3 \Rightarrow f'(1) = 3(8)(2)\left(\dfrac{1}{2}\right) + \left(-\dfrac{1}{2}\right)(8) = 24 - 4 = 20$

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$f(x) = \sin x \cos 2x \Rightarrow f\left(\dfrac{\pi}{2}\right) = \sin\dfrac{\pi}{2}\cos\pi = -1$

$f\left(\dfrac{\pi}{4}\right) = \sin\dfrac{\pi}{4}\cos\dfrac{\pi}{2} = 0 \Rightarrow \dfrac{f\left(\frac{\pi}{2}\right) - f\left(\frac{\pi}{4}\right)}{\frac{\pi}{2} - \frac{\pi}{4}} = \dfrac{-4}{\pi}$

$g(x) = \sin^4 x - \cos^4 x = (\sin^2 + \cos^2)(\sin^2 x - \cos^2 x) = -\cos 2x$

$g\left(\dfrac{\pi}{4}\right) = -\cos\left(\dfrac{\pi}{2}\right) = 0, \quad g\left(\dfrac{\pi}{2}\right) = -\cos\pi = +1$

$\Rightarrow \dfrac{g\left(\frac{\pi}{2}\right) - g\left(\frac{\pi}{4}\right)}{\frac{\pi}{2} - \frac{\pi}{4}} = \dfrac{1}{\frac{\pi}{4}} = \dfrac{4}{\pi}$

$\Rightarrow \dfrac{\frac{-4}{\pi}}{\frac{4}{\pi}} = -1$ برابر

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$$\lim_{x\to 0}\frac{\sin^{n}(x^{۲})\times ۲x\sin^{n-۱}(x^{۲})\times \cos x^{۲}}{(۲\sin^{۲}\frac{x}{۲})^{m}} \xrightarrow{\text{هم ارزی}} \lim_{x\to 0}\frac{(x^{۲})^{n}\times n\times (x^{۲})^{n-۱}\times ۲x}{(۲(\frac{x}{۲})^{۲})^{m}} = \frac{(x)^{۲n+۲n-۲+۱}\times ۲n}{(x)^{۲m}\times\frac{۱}{۲}}$$

$$= \frac{۲nx^{۴n-۱}}{\frac{۱}{۲^{m}}\times x^{۲m}} = ۲^{m+۱}\times n\times x^{۴n-۲m-۱} \Rightarrow ۴n-۲m-۱=۰ \to n=\frac{۲m+۱}{۴}$$

$$\Rightarrow ۲^{m+۱}\times\frac{۲m+۱}{۴}=۳۲\sqrt{۲} \to m=\frac{۷}{۲},\ n=۲ \to \boxed{۲m+n=۹}$$

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$$\frac{f(۰)-f(-۱)}{۰-(-۱)}=\frac{۱-۸(-a+۱)}{۱}=۸a-۷=-۱۱\to ۸a=-۴\to \boxed{a=-\frac{۱}{۲}}$$

$$f'(x)=۳(x^{۲}+۱)^{۲}(۲x)(ax+۱)+a(x^{۲}+۱)^{۳} \xrightarrow{-۲a=۱} f'(۱)=۳(۲)^{۲}(۲)(\frac{۱}{۲})+-\frac{۱}{۲}(۲)^{۳}$$

$$f'(۱)=۱۲-۴=\boxed{۸}$$