$g(x) = f(\tan^2 x + \sqrt{2}\cos x)$ $\xrightarrow[\tan'(a) = \sec^2(a) \times a']{\cos'(a) = -a'\sin a}$ $g'(\frac{\pi}{4}) = 2\tan\frac{\pi}{4} \times \frac{1}{\cos^2\frac{\pi}{4}} + f'(2)(-\sqrt{2}\sin\frac{\pi}{4}) = \sqrt{3}$

$\to 3f'(2) = \sqrt{3} \to f'(2) = \frac{\sqrt{3}}{3}$

۲- $f(x) = \frac{8+\cos^3 x}{4-\cos^2 x} = \frac{(\cos x+2)(\cos^2 x - 2\cos x + 4)}{(2+\cos x)(2-\cos x)} = \cos x - 2 \to y' = -\sin x$

$g(x) = \frac{2}{2-\cos x} \to g'(x) = \frac{-2\sin x}{(\cos x - 2)^2}$

$f'(\frac{7\pi}{6}) - 2g'(\frac{7\pi}{6}) = -\sin\frac{7\pi}{6} + \frac{4\sin\frac{7\pi}{6}}{(\cos\frac{7\pi}{6}-2)^2} = \frac{3\sin\frac{7\pi}{6}}{(\cos\frac{7\pi}{6}-2)^2} = \frac{21+8\sqrt{3}}{3}$

۳- $f(x) = |4x-3|\sqrt{ax}$

$\xrightarrow{\frac{3}{4}^+} (4x-3)\sqrt{ax} \to f'(x) = \frac{4a}{2\sqrt{ax}} = \frac{2a}{\sqrt{ax}}$

$\xrightarrow{\frac{3}{4}^-} (-4x+3)\sqrt{ax} \to f'(x) = \frac{-4a}{\quad}$

ج

۴- $f(2) + f'(2) = -1$

$y = -ax + 2$

مماس در نقطه‌ی ۲ طول ۲ $\to \begin{cases} -2a+2 = f(2) \\ -a = f'(2) \end{cases} \to -5a+2 = -1 \to 2 = 5a - a = 4a \to a = \frac{1}{2} \to f'(2) = -\frac{1}{2}$

۵- $7y - x = b$

مماس ۲ $\to \frac{ax-1}{3x+1} = \frac{x+b}{7} \to 7ax - 7 = 3x^2 + 12x + b \to 3x^2 + (12-7a)x + 12 = 0$

$\frac{ax-1}{3x+1}$

نقطه
تلاقی مضاعف است $\to \Delta = 0 \to 3\underbrace{(x-2)^2}_{x^2-4x+4} = 0 \to -3\times 4 = 12 - 7a$

$a = 4$

۶- $f(x) = x^3 + ax - b \to$ مماس در $x=2$ بر محور طول‌ها $\to \begin{cases} f(2) = 0 \\ f'(2) = 0 \end{cases} \to 3x^2 + a \to 12 + a = 0 \to a = -12$

$8 + 2a - b = 0 \to -16 - b = 0 \to b = -16$ $\Big\}$ $b - a = -4$

[illegible]

ج

-۸

$f^{-1}(9)$: معادله $x=2$ ؟ $\to \frac{\sqrt{x}+1}{\sqrt{x}-1}=2 \to 2\sqrt{x}-2=\sqrt{x}+1 \to \sqrt{x}=3 \to x=9$

$f'(9)=\frac{(\frac{1}{2\sqrt{x}})(\sqrt{x}-1)-(\frac{1}{2\sqrt{x}})(\sqrt{x}+1)}{(\sqrt{x}-1)^2}=\frac{\frac{1}{6}\times2-\frac{1}{6}\times4}{4}=\frac{\frac{1}{3}-\frac{2}{3}}{4}=-\frac{1}{12}\xrightarrow{\text{معکوس}}-12$

-۹

آهنگ متوسط تغییر $=-11 \to \frac{f(0)-f(-1)}{0-(-1)}=-11 \to \frac{1-8(-a+1)}{1}=-11 \to 8a-7=-11 \to 8a=-4 \to a=-\frac{1}{2}$

$\to f(x)=(x^2+1)^3(\underset{-\frac{1}{2}}{a}x+1) \to f'(x)=3(2x)(x^2+1)^2(-\frac{x}{2}+1)-\frac{1}{2}(x^2+1)^3$

$\to f'(1)=3\times\cancel{2}\times4\times\cancel{\frac{1}{2}}-4=8$

$f(x)=\sin x\cos x \to \begin{cases} f(\frac{a}{2})=\sin\frac{a}{2}\cos a=-1 \\ f(\frac{a}{4})=\frac{\sqrt{2}}{2}\times0=0 \end{cases} \to$ آهنگ متوسط تغییرات $=\frac{0-(-1)}{\frac{a}{4}-\frac{a}{2}}$ ①

$f(x)=\sin^4x-\cos^4x=(\sin^2x-\cos^2x)(\sin^2x+\cos^2x)=\sin^2x-\cos^2x=-\cos 2x \to \begin{cases} f(\frac{a}{2})=1 \\ f(\frac{a}{4})=0 \end{cases}$

آهنگ متوسط تغییرات $=\frac{-1}{\frac{a}{4}-\frac{a}{2}}$ ②

$\to \frac{①}{②}=\frac{\frac{+1}{\frac{a}{4}-\frac{a}{2}}}{\frac{-1}{\frac{a}{4}-\frac{a}{2}}}=-1$

طاهر رجبی