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$g(x)=f(\tan^2 x+\sqrt{2}\cos x)\xrightarrow[\tan'(a)=\sec^2(a)\times a']{\cos'(a)=-a'\sin a} g'(\frac{\pi}{4})=2\tan\frac{\pi}{4}\times\frac{1}{\cos^2\frac{\pi}{4}}+f'(2)(-\sqrt{2}\sin\frac{\pi}{4})=\sqrt{3}$

$\to 3f'(2)=\sqrt{3}\to f'(2)=\frac{\sqrt{3}}{3}$

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$f(x)=\frac{8+\cos^3 x}{4-\cos^2 x}=\frac{(\cos x+2)(\cos^2 x-2\cos x+4)}{(2+\cos x)(2-\cos x)}=\cos x-2\to y'=-\sin x$

$g(x)=\frac{2}{2-\cos x}\to g'(x)=\frac{-2\sin x}{(\cos x-2)^2}$

$f'(\frac{7\pi}{6})-2g'(\frac{7\pi}{6})=\frac{-\sin\frac{7\pi}{6}+4\sin\frac{7\pi}{6}}{(\cos\frac{7\pi}{6}-2)^2}=\frac{3\sin\frac{7\pi}{6}}{(\cos\frac{7\pi}{6}-2)^2}=\frac{21+8\sqrt{3}}{3}$

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$f(x)=|4x-3|\sqrt{ax}$

$\frac{3}{4}^+$: $(4x-3)\sqrt{ax}\to f'(x)=\frac{4a}{2\sqrt{ax}}=\frac{2a}{\sqrt{ax}}$

$\frac{3}{4}^-$: $(-4x+3)\sqrt{ax}\to f'(x)=\frac{-4a}{}$

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$f(2)+f'(2)=-1$

$y=-ax+2$

مماس در نقطه‌ای به طول ۲ $\to \begin{cases}-2a+2=f(2)\\ -a=f'(2)\end{cases}\to -3a+2=-1\to 2=5a-a=4a\to a=\frac{1}{2}\to f'(2)=-\frac{1}{2}$

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$7y=x+b$

مماس در ۲ $\to \frac{ax-1}{2x+1}=\frac{x+b}{7}\to 7ax-7=2x^2+12x+b\to 2x^2+(12-7a)x+12=0$

$\frac{ax-1}{2x+1}$

تماس تنها مماس است $\to \Delta=0\to 2(x-2)^2=0\to -2\times 4=12-7a$

x^2-4x+4

$a=4$

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$f(x)=x^3+ax-b\to$ مماس در $x=2$ بر محور xها $\to f(2)=0,\ f'(2)=0\to 3x^2+a\to 12+a=0\to a=-12$

$8+2a-b=0\to -16-b=0\to b=-16$

$b-a=-4$

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[illegible]

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$f^{-1}(x)$ ؟ معادل $x=2$ ؟ $\to \frac{\sqrt{x}+1}{\sqrt{x}-1}=2 \to 2\sqrt{x}-2=\sqrt{x}+1 \to \sqrt{x}=3 \to x=9$

$f'(9)=\frac{\left(\frac{1}{2\sqrt{x}}\right)(\sqrt{x}-1)-\left(\frac{1}{2\sqrt{x}}\right)(\sqrt{x}+1)}{(\sqrt{x}-1)^2}=\frac{\frac{1}{6}\times2-\frac{1}{6}\times4}{4}=\frac{\frac{1}{3}-\frac{2}{3}}{4}=-\frac{1}{12}\xrightarrow{\text{معکوس}}-12$

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آهنگ تغییر متوسط $=-11 \to \frac{f(0)-f(-1)}{0-(-1)}=-11 \to \frac{1-8(-a+1)}{1}=-11 \to 8a-7=-11 \to 8a=-4 \to a=-\frac{1}{2}$

$\to f(x)=(x^2+1)^3\left(\underset{-\frac{1}{2}}{a}x+1\right) \to f'(x)=3(2x)(x^2+1)^2\left(-\frac{x}{2}+1\right)-\frac{1}{2}(x^2+1)^3$

$\to f'(1)=3\times2\times\cancel{4}\times4\times\frac{\cancel{1}}{2}-4=8$

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$f(x)=\sin x\cos x \to \begin{cases} f(\frac{a}{2})=\sin\frac{a}{2}\cos a=-1 \\ f(\frac{a}{4})=\frac{\sqrt{2}}{2}\times0=0 \end{cases} \to$ آهنگ تغییرات $=\frac{0-(-1)}{\frac{a}{4}-\frac{a}{2}}$ (1)

$f(x)=\sin^4x-\cos^4x=(\sin^2x-\cos^2x)(\sin^2x+\cos^2x)=\sin^2x-\cos^2x=-\cos2x \to \begin{cases} f(\frac{a}{2})=1 \\ f(\frac{a}{4})=0 \end{cases}$

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آهنگ تغییرات $=\frac{-1}{\frac{a}{4}-\frac{a}{2}}$ (2)

$\to \frac{(1)}{(2)}=\frac{\frac{+1}{\frac{a}{4}-\frac{a}{2}}}{\frac{-1}{\frac{a}{4}-\frac{a}{2}}}=-1$

طاها رحیمی

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$$f - 2g\left(\frac{7\pi}{4}\right) = \frac{(2+\cancel{\cos x})(\cos^2 x - 2\cos x + 4)}{(2-\cos x)(2+\cancel{\cos x})} - \frac{8}{2-\cos x} = \frac{\overset{\cos x(\cancel{\cos x} - 2)}{\cos^2 x - 2\cos x}}{2 - \cancel{\cos x}} = -\cos x$$

$$(f-2g)' = -(\cos x)' = +\sin x \rightarrow \sin\left(\frac{7\pi}{4}\right) = \boxed{-\frac{1}{2}}$$

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$$x \rightarrow \frac{3}{4}^{+} \rightsquigarrow f(x) = 4x - 3\sqrt{\frac{3a}{4}} \xrightarrow[\text{صفرشونده}]{\text{مشتق عبارت}} f'(x) = \frac{4\sqrt{3a}}{2} = 2\sqrt{3a}$$

$$x \rightarrow \frac{3}{4}^{-} \rightsquigarrow f(x) = -4x + 3\sqrt{\frac{3a}{4}} \xrightarrow[\text{صفرشونده}]{\text{مشتق عبارت}} f'(x) = \frac{-4\sqrt{3a}}{2} = -2\sqrt{3a}$$

$$4\sqrt{3a} = 2\sqrt{6} \rightarrow \sqrt{3a} = \frac{\sqrt{6}}{2} \rightarrow \boxed{a = \frac{1}{2}}$$

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$$y = -ax + 2 \rightarrow f(4) = -4a + 2 \quad , \quad f'(4) = -a$$

$$-4a + 2 - a = -1 \rightarrow -5a = -3 \rightarrow a = +\frac{3}{5} \rightarrow \boxed{f'(4)} = \boxed{-\frac{3}{5}} = -a$$

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$$\lim_{x\to 0} \frac{\sin^n(x^2) \times 2x \sin^{n-1}(x^2) \times \cos x^2}{\left(2\sin^2\frac{x}{2}\right)^m} \xrightarrow{\text{هم ارزی}} \lim_{x\to 0} \frac{(x^2)^n \times n \times (x^2)^{n-1} \times 2x}{\left(2\left(\frac{x}{2}\right)^2\right)^m} = \frac{(x)^{2n+2n-2+1} \times 2n}{(x)^{2m} \times \frac{1}{2}}$$

$$= \frac{2nx^{4n-1}}{\frac{1}{2^m} \times x^{2m}} = 2^{m+1} \times n \times x^{4n-2m-1} \rightarrow 4n - 2m - 1 = 0 \rightarrow n = \frac{2m+1}{4}$$

$$\rightarrow 2^{m+1} \times \frac{2m+1}{4} = 32\sqrt{2} \rightarrow m = \frac{7}{2} \; , \; n = 2 \rightarrow \boxed{2m + n = 9}$$