امین جلالی

۱۹٫۲۵ آفرین

۱

$$g'(x) = (۲\tan x\,(1+\tan^۲ x) - \sqrt{۲}\sin x)\, f'(\tan^۲ x + \sqrt{۲}\cos x)$$

$$\longrightarrow g'(\tfrac{\pi}{۴}) = f'(۲) \times (۲\times۲-1) = \boxed{\frac{\sqrt{۳}}{۳}}$$

۵

۲

$$f(x) = \frac{(\cos x+۲)(\cos^۲ x+۴-۲\cos x)}{(۲-\cos x)(\cos x+۲)} = \frac{\cos^۲ x-۲\cos x+۴}{۲-\cos x}$$

$$\longrightarrow f-۲g = \frac{\cos^۲ x-۲\cos x}{۲-\cos x} = \frac{-\cos x\,(۲-\cos x)}{۲-\cos x}$$

$$= -\cos x \longrightarrow f'(x) - ۲g'(x) = \sin x \longrightarrow f'(\tfrac{۷\pi}{۶}) - ۲g'(\tfrac{۷\pi}{۶})$$

$$= \sin\frac{۷\pi}{۶} = \boxed{-\frac{1}{۲}}$$

۵

$$x \to \frac{3}{2}^{+} \to (2x-3)\sqrt{ax} \Rightarrow f'_{+}\left(\frac{3}{2}\right) = 2\left(\sqrt{\frac{3a}{2}}\right) + \cancel{\frac{a}{2\sqrt{\frac{3a}{2}}}(0^{+})}$$

$\frac{3}{2}$

$$x \to \frac{3}{2}^{-} \to (-2x+3)\sqrt{ax} \Rightarrow f'_{-}\left(\frac{3}{2}\right) = -2\left(\sqrt{\frac{3a}{2}}\right) + \cancel{\frac{a}{2\sqrt{\frac{3}{2}a}}(0^{-})}$$

$$\Longrightarrow f'_{+}\left(\frac{3}{2}\right) - f'_{-}\left(\frac{3}{2}\right) = 2\sqrt{6} \Rightarrow 2\sqrt{\frac{3a}{2}} - \left(-2\sqrt{\frac{3a}{2}}\right) = 2\sqrt{6}$$

$$\overset{4}{\cancel{8}}\sqrt{\frac{3a}{2}} = 2\sqrt{6} \longrightarrow \boxed{a = \frac{1}{2}}$$

2

$f(4) = 2-4a$, $f'(4) = -a$ $\Rightarrow 2-4a-a=-1 \Rightarrow -5a=-3 \Rightarrow \boxed{a=\frac{3}{5}} \Rightarrow f'(4) = -0/6$ ۲ ۱۴ ج

$7y = x+5 \xrightarrow{x=y} 6x=5 \to x=\frac{5}{6} \Rightarrow y' = \frac{a+3}{(3x+1)^2} = \frac{1}{7}$ ۱۵ ب

$\xrightarrow{x=\frac{5}{6}} a+3 = \frac{7}{4} \to \boxed{a=-\frac{5}{4}}$ ۱/۲۵

$f'(2)=0 \Rightarrow 3x^2+a=0 \Rightarrow 12+a=0 \Rightarrow \boxed{a=-12}$ ۱۶ ج

$f(2)=0 \Rightarrow 8-24-b=0 \Rightarrow \boxed{b=-16}$

$\Rightarrow b-a = \boxed{-4}$ ج ۲

$f'(x) = 2ax\sin^{n-1}(x^2).\cos(x^2) \Rightarrow \lim_{x\to 0} \frac{f(x).f'(x)}{(1-\cos x)^m}$ ۱۷ ب

$= \lim_{x\to 0} \frac{\sin^n(x^2) . 2nx\cos(x^2)\sin^{n-1}(x^2)}{(1-\cos x)^m}$ ۲

$(1-\cos x) \sim \frac{x^2}{2}$

$n=2 ,\ m=\cancel{4}\ 2$ $2m+n=\boxed{4}$

$\xrightarrow{\text{هم ارزی}} \lim \frac{2^{m+1} \times n \times n^{4x-1}}{x^{2m}} = 32\sqrt{2} \Rightarrow 2m = 4n-1-m = 2n-1$

$\sqrt{xy} - y = \sqrt{x}+1 \to \sqrt{xy}-\sqrt{x} = 1+y \Rightarrow \sqrt{x}(y-1) = 1+y$ ۱۸ ج

$\Rightarrow \sqrt{x} = \frac{1+y}{y-1} \Rightarrow y = \left(\frac{1+x}{-1+x}\right)^2 \Rightarrow y' = 2\left(\frac{x+1}{x-1}\right)\left(\frac{1(x-1)-1(x+1)}{(x-1)^2}\right)$

$\Rightarrow$ شیب‌ماس $= 2(3)(-2) = \boxed{-12}$ ج ۲

۹

$$\frac{f(0)-f(-1)}{0-(-1)}=11=\frac{1+8a-8}{1}\rightarrow 8a-7=-11\rightarrow a=-\frac{1}{2}$$

$$f'(x)=3(x^2+1)(2x)(-\frac{1}{2}x+1)-\frac{1}{2}(x^2+1)^3\Rightarrow f'(1)=\boxed{8}$$

۱۰

$$\frac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}\xrightarrow{\text{آهنگ متوسط}} f_1(x)=\frac{\sin\frac{\pi}{4}\cos\frac{\pi}{4}-\sin\frac{\pi}{2}\cos\frac{\pi}{2}}{\frac{\pi}{4}}=\boxed{-\frac{1}{\frac{\pi}{2}}}$$

$$f_2(x)=(\sin^2 x-\cos^2 x)(\sin^2 x+\cos^2 x)$$

$$=\sin^2-\cos^2\xrightarrow[\text{متوسط}]{\text{آهنگ}}\frac{-\sin^2\frac{\pi}{2}-\cos^2\frac{\pi}{2}-\sin^2\frac{\pi}{4}+\cos^2\frac{\pi}{4}}{\frac{\pi}{4}}$$

$$=\boxed{\frac{1}{\frac{\pi}{2}}}$$

(-۱) = نسبت ⟸

۵

$$\frac{ax-1}{3x+1} = \frac{x}{7} + \frac{5}{7} \longrightarrow 7ax - 7 = 3x^2 + x + 15x + 5$$

$$3x^2 + x(16-7a) + 12 = 0 \xrightarrow{\Delta = 0} (16-7a)^2 - 12^2 = (16-7a-12)(16-7a+12)$$

$$\boxed{a = \frac{4}{7}} \qquad \boxed{a = 4}$$

$$\boxed{a=4} \longrightarrow 3x^2 - 12x + 12 \longrightarrow \boxed{x = 2} \checkmark$$

$a = \frac{4}{7} \longrightarrow 3x^2 + 12x + 12 \longrightarrow x = -2$ ناحیه اول ✗