شماره تکلیف : ۲۳ — سمی دستجانی

۲۰ آفرین

① $g'(x) = (2\cos x(1+\cos^2 x) - \sqrt{2}\sin x)\, f'(\cos^2 x + \sqrt{2}\cos x)$ ۲

$g'(\frac{\pi}{4}) = (2\times 1\times 2 - 1) f'(1+1) = 3 f'(2) = \sqrt{3} \Rightarrow f'(2) = \frac{\sqrt{3}}{3}$

② $f(x) = \frac{(\cancel{\cos x+2})(\cos^2 x - 2\cos x + 4)}{(2-\cos x)(\cancel{2+\cos x})} = \frac{\cos^2 x - 2\cos x + 4}{2-\cos x}$ ۲

$(f-2g)'(x) = \left(\frac{\cos^2 x - 2\cos x + 4}{2-\cos x} - \frac{4}{2-\cos x}\right)' = \left(\frac{\cos x(\cancel{\cos x - 2})}{-(\cancel{2-\cos x})}\right)' = \sin x$

$\Rightarrow (f-2g)'(\frac{7\pi}{6}) = \sin\frac{7\pi}{6} = \boxed{-\frac{1}{2}}$

③ $f'_+(\frac{3}{4}) = ((4x-3)\sqrt{ax})' = 4\sqrt{ax}\Big|_{\frac34} = 4\times\frac{\sqrt{3a}}{2} = 2\sqrt{3a}$ ۲

ترم صفرکننده

$f'_-(\frac{3}{4}) = (-(4x-3)\sqrt{ax})' = -4\sqrt{ax}\Big|_{\frac34} = -4\times\frac{\sqrt{3a}}{2} = -2\sqrt{3a}$

$\Rightarrow 2\sqrt{3a} - (-2\sqrt{3a}) = 4\sqrt{3a} = \overset{2}{\cancel{4}}\sqrt{9} \Rightarrow \boxed{a = \frac{1}{2}}$

④ $y = -ax + 2 \Rightarrow y = -4a + 2 \Rightarrow f(4) = -4a + 2$ ۲

$f'(4) = -a$ ⊕ $-4a + 2 - a = -1$

$-5a = -3 \Rightarrow \boxed{a = 0/6} \Rightarrow f'(4) = -0/6$

⑤ $y = \frac{1}{7}x + \frac{5}{7} \Rightarrow \frac{ax-1}{3x+1} = \frac{1}{7}x + \frac{5}{7} \Rightarrow 7ax - 7 = 3x^2 + 16x + 5$

$\Rightarrow 3x^2 + (16-7a)x + 12 = 0 \Rightarrow \Delta = 0 \Rightarrow (16-7a)^2 - 144 = 0 \Rightarrow$

$16 - 7a = 12 \rightarrow a = \frac{4}{7}$ غ ق ق

$16 - 7a = -12 \rightarrow a = 4$

مشتق آن نیز صفر است $6x + (\underbrace{\cancel{16-7a}}_{-12}) = 0 \Rightarrow \boxed{x = 2}$ ۲

برای $a = 4$ مماس در نتیجه قابل قبول است

$f(2) = 8 + 2a - b = 0 \Rightarrow b = -16$

$f'(2) = 3 \times 4 + a = 0 \Rightarrow \boxed{a = -12}$ $\Bigg\} \rightarrow b - a = -16 + 12 = \boxed{-4}$ (9)

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$f'(x) = n \sin^{n-1}(x^2) \times 2x \cos(x^2)$ (7)

$f(x) f'(x) = \dfrac{\sin^n(x^2)}{\sin^{n-1}(x^2) \times \sin(x^2)} \times n \sin^{n-1}(x^2) \times 2x \cos(x^2) \Rightarrow$

$f(x) f'(x) = \sin^{2n-2}(x^2) \times \sin(2x^2) \times x \times n$

$$\lim_{x \to 0} \frac{\sin^{2n-2}(x^2) \cdot \sin(2x^2) \times n \times x}{\left(2\sin^2 \frac{x}{2}\right)^m} \overset{\text{هم‌ارزی}}{=} \lim_{x \to 0} \frac{x^{4n-4} \times 2x^2 \times n \times x}{\left(\frac{x^2}{2}\right)^m} =$$

$$\frac{2^m \times 2n \times x^{4n-1}}{x^{2m}} = 16\sqrt{2} \Rightarrow \begin{cases} 2^m \times n = 2^{\frac{9}{2}} \Rightarrow n = 2^z \Rightarrow \\ 2^{m+z} = 2^{\frac{9}{2}} \Rightarrow \boxed{m + z = \frac{9}{2}} \\ 2m = 4n - 1 \\ \boxed{2m = 2^{z+2} - 1} \end{cases} \ominus \Rightarrow 2^{z+2} + 2z = 10 \Rightarrow \boxed{z = 1}$$

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$\Rightarrow n = 2^1 = 2 \quad , \quad m = \frac{7}{2} = 3/5$

$2m + n = 7 + 2 = \boxed{9}$

$\dfrac{\sqrt{x} + 1}{\sqrt{x} - 1} = 2 \Rightarrow \sqrt{x} + 1 = 2\sqrt{x} - 2 \Rightarrow \sqrt{x} = 3 \Rightarrow \boxed{x = 9}$ (8)

$f'(x) = \dfrac{-2 \times \frac{1}{2\sqrt{x}}}{(\sqrt{x} - 1)^2} \Rightarrow f'(9) = \dfrac{-\frac{1}{3}}{4} = -\dfrac{1}{12} \Rightarrow (f^{-1})'(2) = \dfrac{1}{f'(9)}$ 3

$= \boxed{-12}$

4

$$f(-1) = 8(-a+1), \quad f(0) = 1 \Rightarrow \text{آهنگ متوسط} = \frac{f(-1)-f(0)}{-1-0} = \frac{-8a+7}{-1} = -11$$

$$\Rightarrow \boxed{a = -\frac{1}{2}}$$

$$f'(x) = 3(x^2+1)^2(2x)\left(-\frac{1}{2}x+1\right) + \left(-\frac{1}{2}\right)(x^2+1)^3 \Rightarrow$$

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$$f'(1) = 3(2)^2(2)\left(\frac{1}{2}\right) + \left(-\frac{1}{2}\right)\times 8 = 12-4 = 8$$

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$$f(x) = \sin x \cos 2x \Rightarrow \begin{cases} f(\frac{\pi}{4}) = 0 \\ f(\frac{\pi}{2}) = -1 \end{cases} \Rightarrow \text{آهنگ متوسط} = \frac{0-(-1)}{\frac{\pi}{4}-\frac{\pi}{2}} = \boxed{-\frac{4}{\pi}}$$

$$g(x) = -\cos 2x \Rightarrow \begin{cases} f(\frac{\pi}{4}) = 0 \\ f(\frac{\pi}{2}) = 1 \end{cases} \Rightarrow \text{آهنگ متوسط} = \frac{0-1}{\frac{\pi}{4}-\frac{\pi}{2}} = \frac{4}{\pi}$$

$$\Rightarrow \frac{-\frac{4}{\pi}}{\frac{4}{\pi}} = \boxed{-1}$$ — نسبت آهنگ متوسط توابع

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