«به نام خدا»

کیوان مکارم نسب   تکلیف شماره 19

$x \to -2^+ \Rightarrow x>-2 \to -x<2 \to 2-x<4 \to \frac{2-x}{2}<2$

$\to \left[\frac{2-x}{2}\right]=1 \to 2+\cancel{a\left[\frac{x+2}{3}\right]}_{0}=2$ ✓ (1)   (1)

$x>-2 \to x+2>0 \to \frac{x+2}{3}>0 \to \left[\frac{x+2}{3}\right]=0$

$x \to -2^- \Rightarrow x<-2 \to -x>2 \to 2-x>4 \to \frac{2-x}{2}>2 \to \left[\frac{2-x}{2}\right]=2$

$x<-2 \to x+2<0 \to \frac{x+2}{3}<0 \to \left[\frac{x+2}{3}\right]=-1$

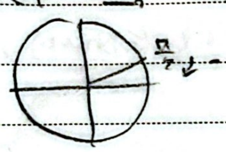
$\to 2(2)+a(-1)=4-a$ ✓ (2) $\xrightarrow[\text{در نقطه } x=-2 \text{ حد دارد}]{(1)=(2)}$ $2=4-a \to a=2$

$\to \left[\frac{a}{3}\right]=\left[\frac{2}{3}\right]=0$ ✓

(2)

$x<\frac{\pi}{6} \to \sin x<\frac{1}{2} \to 2\sin x<1 \to 2\sin x-1<0$

$[2\sin x-1]=-1$ ✓



(3)

$y=8x^3-x \to \lim_{x\to-\frac{1}{2}}[8x^3-x]=\lim_{x\to-\frac{1}{2}}\left[8(-\tfrac{1}{2})^3-(-\tfrac{1}{2})\right]$

$\to \lim_{x\to-\frac{1}{2}}\left[-8(\tfrac{1}{8})+\tfrac{1}{2}\right]=\lim_{x\to-\frac{1}{2}}\left[\overset{-\frac{1}{2}}{\cancel{-1+\tfrac{1}{2}}}\right]=-1$ ✓

(4)

$x<-\frac{1}{2} \to x^2>\frac{1}{4} \to \frac{1}{x^2}<4 \to \frac{3}{x^2}<12 \to \left[\frac{3}{x^2}\right]=11$ (1)

$x<-\frac{1}{2} \to x^2>\frac{1}{4} \to \frac{1}{x^2}<4 \to \frac{2}{x^2}<8 \to -\frac{2}{x^2}>-8 \to \left[\frac{-2}{x^2}\right]=-8$ (2)

$\xrightarrow{(1),(2)} \lim_{x\to(-\frac{1}{2})^-}\frac{10x-5+11}{16x-(-8)}=\lim_{x\to(-\frac{1}{2})^-}\frac{10x+6}{16x+8}=\frac{-5+6}{0^+}=\frac{1}{0^+}=+\infty$ ✓

(5)

$x\to1^+ \to [x]=1 \to \lim_{x\to1^+}\frac{\sin^2\pi x}{1+\cos\pi x} \xrightarrow{\cos^2\alpha=\frac{1+\cos2\alpha}{2}} \lim_{x\to1^+}\frac{\sin\pi x\,\sin\pi x}{2\cos^2\frac{\pi x}{2}}$

$\Rightarrow \lim_{x\to1^+}\frac{\cancel{2}\sin\frac{\pi x}{2}\,\cancel{\cos\frac{\pi x}{2}}\,2\sin\frac{\pi x}{2}\,\cancel{\cos\frac{\pi x}{2}}}{\cancel{2}\,\cancel{\cos\frac{\pi x}{2}}\cdot\cancel{\cos\frac{\pi x}{2}}}=\lim_{x\to1^+}2\sin^2\frac{\pi x}{2}=2\sin^2\frac{\pi}{2}=2$ ✓

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(6)

$$\lim_{x\to -1}\frac{\sqrt{2x+3}-\sqrt{3x+4}}{1+\sqrt[3]{x}} \xrightarrow[\text{مزدوج}]{\frac{0}{0}} \lim_{x\to -1}\frac{\left(\sqrt{2x+3}-\sqrt{3x+4}\right)\left(\sqrt{2x+3}+\sqrt{3x+4}\right)}{\left(1+\sqrt[3]{x}\right)\times\left(\sqrt{2x+3}+\sqrt{3x+4}\right)}$$

$$=\lim_{x\to -1}\frac{2x+3-3x-4}{\left(1+\sqrt[3]{x}\right)(2)} = \lim_{x\to -1}\frac{-(x+1)}{\left(1+\sqrt[3]{x}\right)(2)}\times\frac{\left(1+\sqrt[3]{x^2}-\sqrt[3]{x}\right)}{\left(1+\sqrt[3]{x^2}-\sqrt[3]{x}\right)} = \lim_{x\to -1}\frac{-\cancel{(x+1)}(3)}{\cancel{(x+1)}(2)} = \boxed{\frac{-3}{2}}$$

(7)

$$\lim_{x\to 1}\frac{2x-7\sqrt{x}+5}{2x-\sqrt{3x+1}}\times\frac{2x+\sqrt{3x+1}}{2x+\sqrt{3x+1}} = \lim_{x\to 1}\frac{\left(2x-7\sqrt{x}+5\right)(2)}{4x^2-3x-1} \qquad \frac{\sqrt{x}=t}{t\to 1}$$

$$\lim_{t\to 1}\frac{\left(2t^2-7t+5\right)(2)}{4t^4-3t^2-1} = \lim_{t\to 1}\frac{\cancel{(t-1)}(2t-5)(2)}{\cancel{(t-1)}(4t+1)} = \lim_{t\to 1}\frac{(2t-5)(2)}{(4t+1)} = \frac{(2)(-3)}{5} = \boxed{\frac{-6}{5}}$$

(8)

$$\lim_{x\to 0}\frac{a+\sqrt{bx+c}}{x}=\frac{1}{4} \longrightarrow$$ با توجه به اینکه اگر $x=0$ آنگاه مخرج صفر می‌شود و حاصل حد برابر یک عدد است، پس صورت هم باید مساوی با صفر شود. پس داریم:

$$(1)\ a+\sqrt{c}=0 \Rightarrow a=-\sqrt{c} \to a^2=c \to a^2-c=0$$

$$\to \lim_{x\to 0}\frac{a+\sqrt{bx+c}}{x}\times\frac{a-\sqrt{bx+c}}{a-\sqrt{bx+c}} = \lim_{x\to 0}\frac{a^2-bx-c}{(x)(2a)} \xrightarrow{a^2-c=0} \lim_{x\to 0}\frac{-b\cancel{x}}{\cancel{x}(2a)}=\frac{1}{4}$$

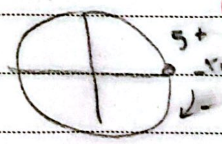
$$\to \frac{-b}{2a}=\frac{1}{4} \to -b=\frac{a}{2} \xrightarrow{(2)} b=-\frac{a}{2}\ ,\ (1)\ \text{طبق} \Rightarrow c=a^2$$

$$\xrightarrow{(1),(2)} \frac{ab}{c} = \frac{-(a)(a)}{2\times a^2} = \boxed{\frac{-1}{2}}$$

(9)

$$(1)\ x\to -2\pi^+ \Rightarrow x>-2\pi \to \frac{x}{\pi}>-2 \to \left[\frac{x}{\pi}\right]=-2$$

$$\to \lim_{x\to -2\pi^+}\frac{4-2k}{0^+}=+\infty \to 4-2k>0 \to k<2\ \boxed{1}$$



$$(2)\ x\to -2\pi^- \Rightarrow x<-2\pi \to \frac{x}{\pi}<-2 \to \left[\frac{x}{\pi}\right]=-3$$

$$\to \lim_{x\to -2\pi^-}\frac{4-3k}{0^-}=+\infty \to 4-3k<0 \to k>\frac{4}{3}\ \boxed{2} \xrightarrow{\boxed{1},\boxed{2}} \frac{4}{3}<k<2$$

$$\xrightarrow{\times -1} -2<-k<-\frac{4}{3} \to [-k]=-2$$

(10)

$$x=8 \to b\sqrt{2+\sqrt[3]{8}}-2b=0 \xrightarrow[\text{صفر باشد}]{\text{مخرج باید}} 8a-b=0 \to 8a=b\ (1)$$

$$\xrightarrow{(1)} \lim_{x\to 8}\frac{8a\sqrt{2+\sqrt[3]{x}}-16a}{ax-8a} \xrightarrow[a]{\div a} \frac{8\sqrt{2+\sqrt[3]{x}}-16}{x-8} \xrightarrow[\ ]{\times 8\sqrt{2+\sqrt[3]{x}}+16} \lim_{x\to 8}\frac{64\left(2+\sqrt[3]{x}\right)-256}{(x-8)(32)} \xrightarrow[t\to 2]{\sqrt[3]{x}=t}$$

$$\lim_{t\to 2}\frac{64t-128}{\left(t^3-8\right)(32)} = \lim_{t\to 2}\frac{64\cancel{(t-2)}}{\cancel{(t-2)}\left(t^2+2t+4\right)(32)} = \lim_{t\to 2}\frac{64}{(12)(32)} = \frac{1}{6}$$