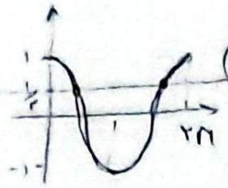
۱۰٫۵ تایم رفع اشکال بگیرید !

نام و نام خانوادگی ...یاسین معمارزاده........ پاسخنامه تشریحی تکلیف شماره ...۱۷... کلاس دوازدهم تجربی B

۱	$8\cos x - \tan^2 x = 1 \Rightarrow 1+\tan^2 x = 8\cos x \Rightarrow \frac{1}{\cos^2} = 8\cos x \Rightarrow 8\cos^3 x = 1 \Rightarrow \cos^3 x = \frac{1}{8}$ $\Rightarrow \cos(x) = \frac{1}{2}$  تعداد جواب = ۲ ۲
۲	$5\sin^2(x) + 3\cos(3x) = -2 \Rightarrow -2 - 5\sin^2(x) = 3\cos(3x) \Rightarrow 5\cos^2 - 7 = 3\cos(3x)$ $\cos(3x) = \cos(x)\cos(2x) - \sin(x)\sin(2x) = \cos^3 x - \sin^2\cos(x) - 2\sin^2\cos(x)$ $= 4\cos^3$ ادامه؟ ۱
۳	$\sin(x)(2\cos(2x) + 1) = 1 \rightarrow \sin(x)(2\cos^2 - 2\sin^2 + 1) = 1$ $\Rightarrow \sin(3\cos^2 - \sin^2) = 1$ ۰٫۲۵
۴	۰
۵	$\cos x \cos\frac{\pi}{4} - \sin x \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}(\cos - \sin) = \frac{1}{\sqrt{2}} \Rightarrow \cos - \sin = \frac{\sqrt{2}}{\sqrt{2}}$ $\Rightarrow \frac{m\sqrt{2}}{\sqrt{2}} - 2\sqrt{2}\ 2\sin\cos$ ادامه؟ ۱

$$(\sin x \cos \tfrac{\pi}{6} + \cos x \sin \tfrac{\pi}{6})(\cos x \cos \tfrac{\pi}{3} + \sin x \sin \tfrac{\pi}{3}) = 1$$

$$\Rightarrow (\tfrac{\sqrt{3}}{2} \sin + \tfrac{1}{2} \cos)(\tfrac{1}{2} \cos + \tfrac{\sqrt{3}}{2} \sin) = 1$$

۶

ادامه؟ .۲۵

$$\tfrac{1}{2} \sin x + \tfrac{\sqrt{3}}{2} \cos x = \tfrac{\sqrt{2}}{2} \Rightarrow \cos \tfrac{\pi}{3} \sin x + \sin \tfrac{\pi}{3} \cos x = \tfrac{\sqrt{2}}{2}$$

$$\Rightarrow \sin(x+\tfrac{\pi}{3}) = \sin \tfrac{\pi}{4} \Rightarrow \begin{cases} x+\tfrac{\pi}{3} = 2k\pi + \tfrac{\pi}{4} \Rightarrow x = 2k\pi - \tfrac{\pi}{12} \\ x+\tfrac{\pi}{3} = 2k\pi + \pi - \tfrac{\pi}{4} \Rightarrow x = 2k\pi + \tfrac{5\pi}{12} \end{cases}$$

۷

مجموع جواب ها؟ ۱٫۵

$$4 \sin x \sin(\tfrac{3\pi}{2} - x) = 4 \sin x(-\cos x) = -4 \sin x \cos x = -2 \sin 2x = 1$$

$$\Rightarrow -\tfrac{1}{2} = \sin(2x) \Rightarrow \sin(2x) = \sin(\tfrac{7\pi}{6}) \Rightarrow \begin{cases} 2x = 2k\pi + \tfrac{7\pi}{6} \\ 2x = 2k\pi + \pi - \tfrac{7\pi}{6} = 2k\pi - \tfrac{\pi}{6} \end{cases}$$

$$\Rightarrow x = k\pi + \tfrac{7\pi}{12}, \quad x = k\pi - \tfrac{\pi}{12}$$

۸

مجموع جواب ها؟ ۱٫۵

$$1+\cos(2\alpha) = 2\cos^2(\alpha) \Rightarrow \underbrace{2\cos^2(\alpha)\, 2\cos^2(2\alpha)}_{\times 2\sin^2(\alpha)}\, 2\cos^2(4\alpha) = \tfrac{1}{8} \times 2\sin^2(2\alpha)$$

$$\alpha = \tfrac{k\pi}{3} \qquad \alpha = \tfrac{k\pi}{5} - \tfrac{\pi}{10}$$

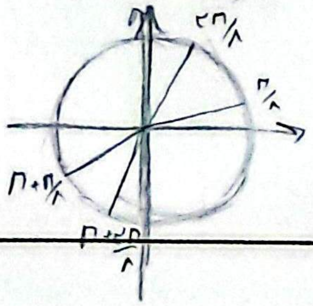
$$\Rightarrow (\underbrace{4\sin^2 \cos^2(\alpha)}_{\sin^2(2\alpha)})(2\cos^2(2\alpha))(2\cos^2(4\alpha)) = \tfrac{1}{4}\sin^2 2\alpha \Rightarrow (\tfrac{1}{2}\sin^2(4\alpha))(2\cos^2(4\alpha))$$

۹

Max ؟ ۱

$$\Rightarrow \tfrac{1}{4}\sin^2(8\alpha) = \tfrac{1}{4}\sin^2(2\alpha) \Rightarrow \sin^2(8\alpha) = \sin^2(2\alpha) \Rightarrow \boxed{\sin 8\alpha = \pm \sin 2\alpha}$$

$$\tan(x) = \frac{1}{\tan(3x)} \Rightarrow \tan(x) = \cot(3x) \Rightarrow x + 3x = \tfrac{\pi}{2} \Rightarrow x = \tfrac{\pi}{8}$$



$$\Rightarrow (\pi + \tfrac{\pi}{8}) + (\pi + \tfrac{3\pi}{8}) = \tfrac{5\pi}{2}$$

۱۰

۱٫۵

۲-

$\cos 3x = 2\cos^2\frac{3x}{2} - 1$ (فرمول طلایی)

$5\sin^2 x + 2\left(2\cos^2\frac{3x}{2} - 1\right) = -2 \rightarrow 5\sin^2 x + 4\cos^2\frac{3x}{2} \cancel{-2} = \cancel{-2}$

حاصل جمع دو عدد نامنفی صفر است پس هر دو صفر بوده اند!

$5\sin^2 x = 0 \rightarrow \sin x = 0 \rightarrow x = k\pi \xrightarrow{-\pi \le x \le \pi} x = \pi$ یا $-\pi$ یا 0

$4\cos^2\frac{3x}{2} = 0 \rightarrow \cos\frac{3x}{2} = 0 \rightarrow \frac{3x}{2} = k\pi + \frac{\pi}{2} \rightarrow x = \frac{2k\pi + \pi}{3}$

$\xrightarrow{-\pi \le x \le \pi} x = -\pi$ یا $-\frac{\pi}{3}$ یا $\frac{\pi}{3}$ یا π

$\boxed{\text{اشتراک دو مجموعه جواب } \{-\pi, \pi\} \text{ است}}$

۳

$\cos 2x = 1 - 2\sin^2 x$ (فرمول طلایی)

$2\sin x(1 - 2\sin^2 x) + \sin x = 1 \rightarrow 3\sin x - 4\sin^3 x = 1$

$\sin 3x = 1 \rightarrow 3x = 2k\pi + \frac{\pi}{2} \rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{6} \xrightarrow{0 \le x \le 2\pi}$

$k = 0 \rightarrow x = \frac{\pi}{6}$, $k = 1 \rightarrow x = \frac{5\pi}{6}$, $k = 2 \rightarrow x = \frac{9\pi}{6}$

$S = \frac{\pi}{6} + \frac{5\pi}{6} + \frac{9\pi}{6} = \frac{15\pi}{6} = \boxed{\frac{5\pi}{2}}$

۴

$\begin{cases} \sin(2x + \frac{\pi}{2}) = \cos 2x \\ \cos(\frac{\pi}{2} + 4x) = -\sin 4x \end{cases} \rightarrow \frac{1}{\cos 2x} + \frac{1}{-\sin 4x} = 0 \rightarrow \frac{\cos 2x - \sin 4x}{\cos 2x(-\sin 4x)} = 0$

$\cos 2x - 2\sin 2x\cos 2x = 0 \rightarrow \cos 2x(1 - 2\sin 2x) = 0$ → $\cos 2x = 0$ ✗ ; $\sin 2x = \frac{1}{2}$ ✓

* اگر $\cos 2x = 0$ باشد مخرج عبارت اولیه صفر می‌شود که غیر قابل قبول است!

$\sin 2x = \frac{1}{2} \rightarrow 2x = 2k\pi + \frac{\pi}{6}$ یا $2k\pi + \frac{5\pi}{6} \rightarrow x = k\pi + \frac{\pi}{12}$ یا $k\pi + \frac{5\pi}{12}$

$\xrightarrow{0 \le x \le \pi} k = 0 \rightarrow x = \frac{\pi}{12}$ یا $\frac{5\pi}{12} \rightarrow \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3}$

$\alpha = \frac{\pi}{3} \rightarrow \tan(2\alpha) = \tan(\frac{2\pi}{3}) = \boxed{-\sqrt{3}}$

۵

$$\cos\left(x+\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{3}}{3} \longrightarrow \cos x - \sin x = \frac{\frac{\sqrt{3}}{3}}{\frac{1}{\sqrt{2}}}$$

$$\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}} \xrightarrow{\text{گویا}} \frac{\sqrt{6}}{3} \longrightarrow \boxed{\cos x - \sin x = \frac{\sqrt{6}}{3}}$$

$$(\cos x - \sin x)^2 = 1 - \sin 2x = \frac{6}{9} \longrightarrow \boxed{\sin 2x = \frac{1}{3}}$$

$$m(\cos x - \sin x) - 3\sqrt{6}(\sin 2x) = \sqrt{6} \longrightarrow \frac{\sqrt{6}}{3}m - 3\sqrt{6}\left(\frac{1}{3}\right) = \sqrt{6}$$

$$\frac{\sqrt{6}}{3}m - \sqrt{6} = \sqrt{6} \longrightarrow \frac{m\sqrt{6}}{3} = 2\sqrt{6} \longrightarrow \boxed{m=6}$$

۶

$$\cos\left(x-\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}-x\right)$$

$$\frac{\pi}{3} - x + x + \frac{\pi}{6} = \frac{\pi}{2} \longrightarrow \cos\left(\frac{\pi}{3}-x\right) = \sin\left(x+\frac{\pi}{6}\right)$$

$$\sin^2\left(x+\frac{\pi}{6}\right) = 1 \longrightarrow \sin\left(x+\frac{\pi}{6}\right) = \pm 1 \longrightarrow x + \frac{\pi}{6} = k\pi + \frac{\pi}{2}$$

$$x = k\pi + \frac{\pi}{3} \xrightarrow{0 \le x \le 2\pi} k=0 \to x = \frac{\pi}{3}, \quad k=1,\ x = \frac{4\pi}{3}$$

معادله ۲ جواب در بازه دارد

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۷ عبارت را در $\frac{1}{2}$ ضرب می‌کنیم.

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

$$\cos 60^\circ \sin x + \sin 60^\circ \cos x = \frac{\sqrt{2}}{2} \longrightarrow \sin\left(x+\frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

$$\begin{cases} x + \frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \longrightarrow x = 2k\pi - \frac{\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0,1 \to x = -\frac{\pi}{12}, \frac{23\pi}{12} \\ x + \frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \longrightarrow x = 2k\pi + \frac{5\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0 \to x = \frac{5\pi}{12} \end{cases}$$

$$S = \frac{23\pi}{12} - \frac{\pi}{12} + \frac{5\pi}{12} = \frac{27\pi}{12} = \boxed{\frac{9\pi}{4}}$$

(۹)

$1+\cos 2x = 2\cos^2 x$

$$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \xrightarrow{?} 8\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha \cos 2\alpha \cos 4\alpha = \frac{\pm 1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha} *$$

$$\underbrace{(\sin\alpha \cos\alpha)}_{\frac{1}{2}\sin 2\alpha} \cos 2\alpha \cos 4\alpha = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{2}\underbrace{(\sin 2\alpha \cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha} \cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha \cos 4\alpha)}_{\sin 8\alpha} = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{\cancel{8}}\sin 8\alpha = \pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \rightarrow \begin{cases} 8\alpha = 2k\pi+\alpha \text{ یا } 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi-\alpha \text{ یا } 2k\pi+\pi+\alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{MAX\ A} k=3 \rightarrow x = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{MAX\ A} k=4 \rightarrow x = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{MAX\ A} k=2 \rightarrow x = \frac{5\pi}{7}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{MAX\ A} k=3 \rightarrow x = \frac{7\pi}{9}$$

$$\rightarrow \boxed{MAX\ A = \frac{8\pi}{9}}$$

* A نمی‌تواند برابر خود π باشد چون طبق * sinα را مخالف صفر در نظر گرفته‌ایم!

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(۸)

$$\sin\left(\frac{3\pi}{2}-x\right) = -\cos x$$

$$4\sin x(-\cos x) = 1 \rightarrow -2(2\sin x\cos x) = 1 \rightarrow \sin 2x = -\frac{1}{2}$$

$$2x = 2k\pi - \frac{\pi}{6} \text{ یا } 2k\pi + \frac{7\pi}{6} \rightarrow x = \overset{2}{k\pi - \frac{\pi}{12}} \text{ یا } \overset{1}{k\pi + \frac{7\pi}{12}}$$

$$\xrightarrow{0 \le x \le 2\pi} \begin{cases} \overset{1}{k = 0, 1} \rightarrow x = \frac{7\pi}{12}, \frac{19\pi}{12} \\ \underset{2}{k = 1, 2} \rightarrow x = \frac{11\pi}{12}, \frac{23\pi}{12} \end{cases}$$

$$S = \frac{7\pi}{12} + \frac{19\pi}{12} + \frac{11\pi}{12} + \frac{23\pi}{12} = \frac{60\pi}{12} = \boxed{5\pi}$$

10

$$\tan 3x = \frac{1}{\tan x} = \cot x \longrightarrow \tan 3x = \tan\left(\frac{\pi}{2} - x\right)$$

$$3x = k\pi + \frac{\pi}{2} - x \longrightarrow 4x = k\pi + \frac{\pi}{2} \rightarrow x = \frac{k\pi}{4} + \frac{\pi}{8}$$

k	4	5	6	7
x	$\frac{9\pi}{8}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{15\pi}{8}$

$$S = \left(\frac{9+11+13+15}{8}\right)\pi = \frac{48\pi}{8} = \boxed{6\pi}$$