①

$8\cos x - \tan^2 x = 1$ $\quad [0, 2\pi]$ بازه

$1+\tan^2 = 8\cos$ $\quad \left(\frac{1}{\cos^2}\right) \longrightarrow 8\cos^3 = 1 \longrightarrow \cos^3 = \frac{1}{8} \to \cos = \frac{1}{2}$ $\quad$ ۲ $\quad x = \frac{\pi}{3}, \frac{5\pi}{3}$

۲ جواب

۱۹٫۵ آفرین

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②

$5\sin^2 x + 2\cos 3x = -2$ $\quad [-\pi, \pi]$

$5 - 5\cos^2 + 2(4\cos^3 - 3\cos) = -2 \longrightarrow 7 = 5\cos^2 - 8\cos^3 + 6\cos - 7$

$8\cos^3 - 5\cos^2 - 6\cos + 7 = 0 = (\cos+1)(8\cos^2 - 13\cos + 7) = 0$ $\quad$ ۲

-1 $\quad \Delta <$ ریشه ندارد

۲ جواب

$\cos = -1 \longrightarrow \pi \longrightarrow [-\pi, \pi] \longrightarrow -\pi, \pi$

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③

$2\sin x \cos 2x + \sin x = 1$ $\quad [0, 2\pi]$

$\sin(2\underset{1-2\sin^2}{\cos 2x} + 1) = 1 \longrightarrow \sin(3 - 4\sin^2) = 1 \longrightarrow 3\sin - 4\sin^3 = \sin 3x$

$\sin 3x = 1 \longrightarrow 3x = 2k\pi + \frac{\pi}{2} \longrightarrow x = \frac{2k\pi}{3} + \frac{\pi}{6} \longrightarrow \frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6}$

$\frac{\pi}{6} + \frac{5\pi}{6} + \frac{9\pi}{6} = \frac{15\pi}{6} = \frac{5\pi}{2}$ $\quad \boxed{\frac{5\pi}{2}}$ $\quad$ ۲

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④

$\frac{1}{\sin\left(\frac{\pi+4x}{2}\right)} + \frac{1}{\cos\left(\frac{\pi+8x}{2}\right)} = 0$ $\quad [0, \pi]$ $\quad \tan 2\alpha = ?$ $\quad$ ۲

$\frac{1}{\cos 2x} - \frac{1}{\sin 4x} = \frac{\sin 4x - \cos 2x}{\cos 2x \sin 4x} \quad \frac{2\sin 2x\cos 2x - \cos 2x}{\cos 2x \sin 2x \cos 2x}$

$\frac{2\sin 2x - 1}{2\sin 2x \cos 2x} = 0 \longrightarrow 2\sin 2x = 1 \longrightarrow \sin 2x = \frac{1}{2} \longrightarrow 2x = 2k\pi + \frac{\pi}{6}, \frac{5\pi}{6}$

$x = k\pi + \frac{\pi}{12}, \frac{5\pi}{12}$

$k\pi + \frac{5\pi}{12} - (k\pi + \frac{\pi}{12}) = \frac{4\pi}{12} = \frac{\pi}{3}$ $\quad \boxed{\tan\frac{2\pi}{3} = -\sqrt{3}}$

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⑤

$m(\cos x - \sin x) - 3\sqrt{6}\sin(2x) = \sqrt{6}$ $\quad \cos(x + \frac{\pi}{4}) = \frac{1}{\sqrt{3}}$ $\quad m = ?$

$\cos - \sin = \frac{\sqrt{2}}{\sqrt{3}}$

$\frac{\cos - \sin}{\sqrt{2}} \longrightarrow (\cos - \sin)^2 = \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2$

$\underset{1}{\cos^2 + \sin^2} - \underset{\sin 2x}{2\sin x\cos x} = \frac{2}{3} \longrightarrow \sin 2x = \frac{1}{3}$ $\quad m\left(\frac{\sqrt{2}}{\sqrt{3}}\right) - \sqrt{6} = \sqrt{6} \to m\left(\frac{\sqrt{2}}{\sqrt{3}}\right) = 2\sqrt{6}$

$\sqrt{2}m = 2\frac{\sqrt{18}}{\sqrt{9}} = m = 2 \times 3 = 6$ $\quad \boxed{m = 6}$ $\quad$ ۲

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⑥

$\sin(x + \frac{\pi}{6})\cos(x - \frac{\pi}{3}) = 1$ $\quad [0, 2\pi]$

$x + \frac{\pi}{6} - \frac{\pi}{2} = x - \frac{\pi}{3} \longrightarrow \cos(x - \frac{\pi}{3}) = \sin(x + \frac{\pi}{6})$ $\quad \sin^2(x + \frac{\pi}{6}) = 1$

$\sin(x + \frac{\pi}{6}) = \pm 1 \longrightarrow x = \frac{\pi}{3}, \frac{4\pi}{3}$ $\quad \boxed{\frac{\pi}{3}, \frac{4\pi}{3}}$ $\quad$ ۲

⑦

$\sin x + \sqrt{۳}\cos x = \sqrt{۲}$ $[-\pi ، ۲\pi]$

$\frac{۱}{۲}\sin x + \frac{\sqrt{۳}}{۲}\cos x = \frac{\sqrt{۲}}{۲} \longrightarrow \sin(x+\frac{\pi}{۳}) = \sin\frac{\pi}{۴}$

$x + \frac{\pi}{۳} = \frac{\pi}{۴} \longrightarrow x = ۲k\pi - \frac{\pi}{۱۲} \longrightarrow x = -\frac{\pi}{۱۲} ، \frac{۲۳\pi}{۱۲}$

۱,۵

$-\frac{\pi}{۱۲} + \frac{۲۳\pi}{۱۲} = \frac{۲۲\pi}{۱۲} = \frac{۱۱\pi}{۶}$

$\frac{۱۱\pi}{۶}$

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⑧

$۴\sin x \sin(\frac{۳\pi}{۲} - x) = ۱$ $[۰ ، ۲\pi]$

$-\cos x$

$-۴\sin x\cos x = ۱ \longrightarrow \sin ۲x = \frac{-۱}{۲} \longrightarrow ۲x = ۲k\pi - \frac{\pi}{۶} ، + \frac{۷\pi}{۶}$

$-۲\sin ۲x$

$x = k\pi - \frac{\pi}{۱۲} ، \frac{۷\pi}{۱۲}$

$\frac{۱۱\pi}{۱۲} + \frac{۲۳\pi}{۱۲} ، \frac{۷\pi}{۱۲} ، \frac{۱۹\pi}{۱۲}$ $\frac{۶۰\pi}{۱۲} = ۵\pi$ ✓ ۵π ۲

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⑨

$(۱+\cos(۲\alpha))(۱+\cos(۴\alpha))(۱+\cos(۸\alpha)) = \frac{۱}{۸}$ max جواب A

$\cos^۲\alpha - \sin^۲\alpha$ $\cos^۲ ۲\alpha - \sin^۲ ۲\alpha$ $\cos^۲ ۴\alpha - \sin^۲ ۴\alpha$

$(۲\cos^۲\alpha)(۲\cos^۲ ۲\alpha)(۲\cos^۲ ۴\alpha) = \frac{۱}{۸} \longrightarrow \cos\alpha\cos ۲\alpha\cos ۴\alpha = \pm\frac{۱}{۸}$

$\sin\alpha\cos\alpha\cos ۲\alpha\cos ۴\alpha = \pm\frac{۱}{۸}\sin\alpha \longrightarrow \sin ۸\alpha = \pm\sin\alpha$

$\frac{\sin ۲\alpha}{۲}$ $\frac{\sin ۴\alpha}{۴}$ $\frac{\sin ۸\alpha}{۸}$

$۸\alpha = ۲k\pi + \alpha \longrightarrow \alpha = \frac{۲k\pi}{۷}$ ۲

$۸\alpha = ۲k\pi - \alpha \Rightarrow \frac{۲k\pi}{۹} \longrightarrow \frac{۸\pi}{۹}$

$۲k\pi + \pi + \alpha \Rightarrow \frac{۲k\pi}{۷}$

$\frac{۸\pi}{۹}$ ✓

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⑩

$\tan(۳x)\tan(x) = ۱$ $[\pi ، ۲\pi]$ $\cot(۳x)\tan(۳x) = ۱$

$\tan(x) = \cot(۳x) \longrightarrow \cot ۳x = \cot(\frac{\pi}{۲} - x)$

$۳x = k\pi + \frac{\pi}{۲} - x \longrightarrow x = \cancel{\frac{۲k\pi + ۲\pi}{۸}} \longrightarrow$ ۲ $\frac{۹\pi}{۸}$ $\frac{۱۱\pi}{۸}$ $\frac{۱۳\pi}{۸}$ $\frac{۱۵\pi}{۸}$

$\frac{k\pi}{۴} + \frac{\pi}{۸}$ $\frac{۴۸\pi}{۸} = ۶\pi$

۶π ✓

دوازدهم [illegible]

⑦ عبارت را در $\frac{۱}{۲}$ ضرب می‌کنیم.

$$\frac{۱}{۲}\sin x + \frac{\sqrt{۳}}{۲}\cos x = \frac{\sqrt{۲}}{۲}$$

$$\cos ۶۰^\circ \sin x + \sin ۶۰^\circ \cos x = \frac{\sqrt{۲}}{۲} \longrightarrow \sin\left(x+\frac{\pi}{۳}\right) = \frac{\sqrt{۲}}{۲}$$

$$\begin{cases} x+\frac{\pi}{۳} = ۲k\pi + \frac{\pi}{۴} \longrightarrow x = ۲k\pi - \frac{\pi}{۱۲} \xrightarrow{-\pi \leq x \leq ۲\pi} k = ۰ , ۱ \rightarrow x = -\frac{\pi}{۱۲} , \frac{۲۳\pi}{۱۲} \\ x+\frac{\pi}{۳} = ۲k\pi + \frac{۳\pi}{۴} \longrightarrow x = ۲k\pi + \frac{۵\pi}{۱۲} \xrightarrow{-\pi \leq x \leq ۲\pi} k = ۰ \rightarrow x = \frac{۵\pi}{۱۲} \end{cases}$$

$$S = \frac{۲۳\pi}{۱۲} - \frac{\pi}{۱۲} + \frac{۵\pi}{۱۲} = \frac{۲۷\pi}{۱۲} = \boxed{\frac{۹\pi}{۴}}$$