۲۰ آفرین

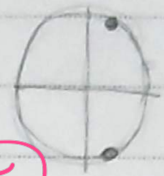
به نام خدا

امیرعلی نورعلی ایران ۱۲ دی

۱۷ الف

۱- $8\cos x - \tan^2 x = 1 \to 8\cos x = \frac{1}{\cos^2 x}$

$\to 8\cos^3 x = 1 \to \cos^3 x = \frac{1}{8} \to \cos x = \frac{1}{2}$



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$\to$ ۲ جواب $\to \frac{\pi}{3}, \frac{5\pi}{3}$

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$5\sin^2 x + 2\cos 3x = -2 \to 5 - 5\cos^2 x + 2(4\cos^3 x - 3\cos x) = -2$

$\to 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0 \to$ جمع ضرایب فرد = زوج

$8\cos^3 x - 5\cos^2 x - 6\cos x + 7 \mid \cos x + 1$

$-8\cos^3 x - 8\cos^2 x \qquad 8\cos^2 x - 13\cos x + 7$

$\to$ ریشه ندارد

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$\hookrightarrow \cos x = -1 \to x = \pi, -\pi$

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$2\sin x \cos 2x + \sin x = 1 \to 2\sin x(1 - 2\sin^2 x) + \sin x = 1$

$\to 3\sin x - 4\sin^3 x = 1 \to 4\sin^3 x - 3\sin x + 1 = 0$

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$\to -\sin(3x) = -1 \to \sin 3x = 1 \to 3x = 2k\pi + \frac{\pi}{2}$

$\to \frac{2k\pi}{3} + \frac{\pi}{6} \to$  $\to$ ۳ تا جواب $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6} \to \frac{15\pi}{6} = \frac{5\pi}{2}$

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$\frac{1}{\sin(\frac{\pi}{2}+2x)} + \frac{1}{\cos(\frac{\pi}{2}+4x)} \to \frac{1}{\cos 2x} = \frac{-1}{\cos(\frac{\pi}{2}+4x)} \to -\cos 2x = \cos(\frac{\pi}{2}+4x)$

$\to \cos(2x - \pi) = \cos(\frac{\pi}{2}+4x) \to 2x - \pi = \frac{\pi}{2} + 4x \to -\frac{3\pi}{2} = 2x$

$\to \cos 4x = 2\sin 2x \cos 2x,\ 2x - \pi = -4x - \frac{\pi}{2} \to \frac{\pi}{2} - 4x \to x = \frac{\pi}{12}$

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$\hookrightarrow \sin 2x = \frac{1}{2} \to x = \frac{\pi}{12}, \frac{5\pi}{12}$, $x = \frac{\pi}{3}$

$\to \boxed{\tan\frac{2\pi}{3} = -\sqrt{3}}$

-۵

$$\cos(x+\frac{\pi}{4})=\frac{1}{\sqrt{3}} \Rightarrow \frac{1}{\sqrt{2}}(\cos x-\sin x)=\frac{1}{\sqrt{3}}$$

$$\Rightarrow \cos x-\sin x=\boxed{\frac{\sqrt{6}}{3}} \Rightarrow m(\cos x-\sin x)-3\sqrt{6}\sin(2x)=\sqrt{6}$$

$$\Rightarrow \frac{m}{3}-3\sin 2x=1$$

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$$(\cos x-\sin x)^2=(\frac{\sqrt{6}}{3})^2 \Rightarrow \cos^2 x+\sin^2 x-\sin 2x=\frac{6}{9}$$

$$\Rightarrow \sin 2x=\frac{3}{9} \longrightarrow \frac{m}{3}-3\times\frac{3}{9}=1 \longrightarrow \frac{m}{3}=2 \longrightarrow m=6$$

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$$\sin(x+\frac{\pi}{6})\cos(x-\frac{\pi}{3})=1$$

$$\Rightarrow (\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x)(\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x)=\frac{\sqrt{3}}{4}\cos^2+\frac{\sqrt{3}}{4}\sin^2+\frac{\sqrt{3}}{4}\sin 2x$$

$$\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4}\sin 2x=1 \Rightarrow \frac{4\sqrt{3}}{3}=1+\sin 2x \Rightarrow \frac{4\sqrt{3}-3}{3}=\sin 2x$$

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$$\Rightarrow \sin 2x=\frac{\sqrt{3}}{2} \Rightarrow 2x=2k\pi+\frac{\pi}{3},\boxed{2k\pi+\frac{2\pi}{3}} \xrightarrow{\times}$$ کار نمی‌اند

زیرا منفی منفی جا نمی‌شود

$$\Rightarrow \boxed{\frac{\pi}{3},\frac{4\pi}{3}} \longrightarrow$$ جواب ها $\longrightarrow$ $\boxed{۱ \text{ جواب}}$

-۷

$$\sin x+\sqrt{3}\cos x=\sqrt{2} \Rightarrow \cancel{2}\sin(x+\frac{\pi}{3})=\frac{\sqrt{2}}{2}$$

$$\Rightarrow x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4},2k\pi+\frac{3\pi}{4} \rightarrow x=2k\pi-\frac{\pi}{12},2k\pi+\frac{5\pi}{12}$$

$-\frac{\pi}{12},$

$$\Rightarrow \frac{5\pi}{12},\frac{23\pi}{12} \Rightarrow \frac{27\pi}{12}=\frac{9\pi}{4}$$

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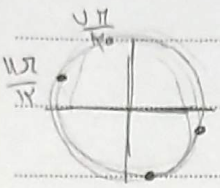
$4\sin x \sin\left(\frac{3\pi}{2} - x\right) = 1$

$\Rightarrow -4\sin x \cos x = 1 \longrightarrow 2\sin 2x = -1 \longrightarrow \boxed{\sin 2x = -\frac{1}{2}}$



$2x = 2k\pi - \frac{\pi}{6} \longrightarrow x = k\pi - \frac{\pi}{12}$

$2x = 2k\pi - \frac{5\pi}{6} \longrightarrow x = k\pi - \frac{5\pi}{12}$

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$\Rightarrow \frac{11\pi}{12} + \frac{7\pi}{12} + \frac{19\pi}{12} + \frac{23\pi}{12} \Rightarrow \frac{60\pi}{12} = 5\pi$

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$(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha) = \frac{1}{8}$

$\Rightarrow (2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha) = \frac{1}{8}$

$\times \frac{4\sin^2\alpha}{4\sin^2\alpha}$

$\Rightarrow 16\,\underbrace{\sin^2\alpha \cos^2\alpha}_{4\sin^2 2\alpha}\,\underbrace{\cos^2 2\alpha \cos^2 4\alpha}_{\sin^2 8\alpha} = \frac{\cancel{4}\sin^2\alpha}{8 \cdot 4}$

$\frac{1}{\cancel{4}}\sin^2 8\alpha = \frac{\sin^2\alpha}{\cancel{4}} \longrightarrow \boxed{\sin^2 8\alpha = \sin^2\alpha} \longrightarrow 8\alpha = k\pi \pm \alpha$

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$\begin{cases} 8\alpha = k\pi + \alpha \longrightarrow 7\alpha = k\pi \longrightarrow \alpha = \frac{k\pi}{7} \\ 8\alpha = k\pi - \alpha \Rightarrow 9\alpha = k\pi \longrightarrow \alpha = \frac{k\pi}{9} \end{cases} \Rightarrow \cancel{\boxed{\max}} = \frac{8\pi}{9}$

π نیست زیرا که در عبارت اولیه کار نمی‌کند

۱۰ـ

$\tan(3x)\tan x = 1 \longrightarrow \boxed{1 - \tan 3x \tan x = 0} \Rightarrow \tan 4x = \frac{\tan x + \tan 3x}{1 - \tan 3x \tan x}$

$\Rightarrow \tan 4x = \frac{\sin 4x}{\cos 4x} \longrightarrow \cos 4x = 0 \longrightarrow 4x \Rightarrow k\pi + \frac{\pi}{2}$

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$\Rightarrow x = \frac{k\pi}{4} + \frac{\pi}{8} \Rightarrow \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8} \Rightarrow \overset{\text{مجموع}}{\frac{48\pi}{8}} = 6\pi$