به نام خدا

سامان نژادکرد — دوازدهم ریاضی B — تکلیف ۱۷

۱ـ

$8\cos x - \tan^2 x = 1 \Rightarrow 8\cos x = 1 + \tan^2 x = \frac{1}{\cos^2 x}$

$\Rightarrow 8\cos^3 x = 1 \Rightarrow \cos x = \frac{1}{2} \Rightarrow x = \begin{cases} 2k\pi + \frac{\pi}{3} \Rightarrow \frac{\pi}{3} \\ 2k\pi - \frac{\pi}{3} \Rightarrow \frac{5\pi}{3} \end{cases}$ در بازه $[0, 2\pi]$

$\{\frac{\pi}{3}, \frac{5\pi}{3}\}$ دو جواب دارد

۲ـ

$5\sin^2 x + 2\cos 3x = -2 \Rightarrow 5(1-\cos^2 x) + 2(4\cos^3 x - 3\cos x) = -2$

$\Rightarrow 5 - 5\cos^2 x + 8\cos^3 x - 6\cos x = -2 \Rightarrow 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0$

$\Rightarrow (\cos x + 1)(8\cos^2 x - 13\cos x + 7) = 0$

$\Rightarrow \begin{cases} \cos x = -1 \Rightarrow \boxed{x = \pi} \text{ } \boxed{x = -\pi} \\ 8\cos^2 x - 13\cos x + 7 = 0 \Rightarrow \Delta < 0 \end{cases}$

۳ـ

$2\sin x\cos 2x + \sin x = 1 \Rightarrow 2\sin x(1 - 2\sin^2 x) + \sin x = 1$

$\Rightarrow 2\sin x - 4\sin^3 x + \sin x = 1 \Rightarrow 4\sin^3 x - 3\sin x + 1 = 0$

$\Rightarrow (\sin x + 1)(4\sin^2 x - 4\sin x + 1) = 0$

$\Rightarrow \begin{cases} \sin x = -1 \Rightarrow x = 2k\pi - \frac{\pi}{2} \Rightarrow x = \frac{3\pi}{2} \\ 4\sin^2 x - 4\sin x + 1 = 0 \Rightarrow \sin x = +\frac{1}{2} \Rightarrow x = 2k\pi + \frac{\pi}{6} \Rightarrow x = \frac{\pi}{6} \\ \quad 2k\pi + \frac{5\pi}{6} \Rightarrow x = \frac{5\pi}{6} \end{cases}$

$x = \{\frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}\}$

۴ـ

$\frac{1}{\sin(\frac{\pi}{2} + 2x)} + \frac{1}{\cos(\frac{\pi}{2} + 4x)} = \frac{1}{\cos 2x} + \frac{-1}{\sin 4x} = \frac{\sin 4x - \cos 2x}{\cos 2x \sin 4x} = 0$

$\Rightarrow 2\sin 2x\cos 2x - \cos 2x = 0 \Rightarrow \begin{cases} \cos 2x = 0 \text{ بشرط تحقق} \\ \sin 2x = \frac{1}{2} \Rightarrow 2x = 2k\pi + \frac{\pi}{6} \Rightarrow x = \frac{\pi}{12} \\ \quad 2k\pi + \frac{5\pi}{6} \Rightarrow x = \frac{5\pi}{12} \end{cases}$

$\to x_2 - x_1 = \frac{5\pi}{12} - \frac{\pi}{12} = \frac{4\pi}{12} = \frac{\pi}{3} = \alpha$

$\to \tan 2\alpha = \tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$

۵ -

$\cos\left(x + \frac{\pi}{4}\right) = \frac{\cos x - \sin x}{\sqrt{2}} = \frac{1}{\sqrt{3}}$

$\to \boxed{\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}} \to (\cos x - \sin x)^2 = 1 - \sin 2x = \frac{2}{3} \to \sin 2x = \frac{1}{3}$

$\to m(\cos x - \sin x) - 3\sqrt{6}\sin 2x = m\left(\frac{\sqrt{6}}{3}\right) - 3\sqrt{6}\sin 2x = \sqrt{6}$

$\to m\left(\frac{\sqrt{6}}{3}\right) - \sqrt{6} = \sqrt{6} \to \boxed{m = 6}$

۶ -

$\sin\left(x + \frac{\pi}{6}\right)\underbrace{\cos\left(x - \frac{\pi}{3}\right)}_{\sin\left(\frac{\pi}{6} + x\right)} = 1 \to \sin^2\left(\frac{\pi}{6} + x\right) = 1$

$$\to \begin{cases} \sin\left(x + \frac{\pi}{6}\right) = 1 \to x + \frac{\pi}{6} = 2k\pi + \frac{\pi}{2} \to x = 2k\pi + \frac{\pi}{3} \\ \sin\left(x + \frac{\pi}{6}\right) = -1 \to x + \frac{\pi}{6} = 2k\pi - \frac{\pi}{2} \to x = 2k\pi - \frac{2\pi}{3} \end{cases}$$

$\to x = \frac{\pi}{3}, \ x = \frac{4\pi}{3} \to$ ۲ جواب

۷ -

$\sin x + \sqrt{3}\cos x = \sqrt{2} \to \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$

$$\to \sin\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2} \to \begin{cases} x + \frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \to x = 2k\pi - \frac{\pi}{12} \\ x + \frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \to x = 2k\pi + \frac{5\pi}{12} \end{cases}$$

$$\to \begin{cases} x = \frac{24k\pi - \pi}{12} \to k = 0, 1 \to x = \left\{-\frac{\pi}{12}, \frac{23\pi}{12}\right\} \\ x = \frac{24k\pi + 5\pi}{12} \to k = 0 \to x = \left\{\frac{5\pi}{12}\right\} \end{cases}$$

$-\pi < x < 2\pi$

$\to -\frac{\pi}{12} + \frac{5\pi}{12} + \frac{23\pi}{12} = \frac{27\pi}{12}$

۸-

$4\sin x \sin(\frac{3\pi}{2}-x)=1$

$\to -4\sin x\cos x=1 \to -2\sin 2x=1 \to \sin 2x=-\frac{1}{2}$

$\to \begin{cases} 2x=2k\pi+\frac{11\pi}{6} \to x=k\pi+\frac{11\pi}{12} \to x=\frac{11\pi}{12},\frac{23\pi}{12} \\ 2x=2k\pi+\frac{7\pi}{6} \to x=k\pi+\frac{7\pi}{12} \to x=\frac{7\pi}{12},\frac{19\pi}{12} \end{cases}$

$\to$ مجموع جوابها: $\frac{11\pi}{12}+\frac{23\pi}{12}+\frac{7\pi}{12}+\frac{19\pi}{12}=\frac{60\pi}{12}=5\pi$

۹-

$(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha)=\frac{1}{8}$

$\to 2\cos^2\alpha \times 2\cos^2 2\alpha \times 2\cos^2 4\alpha=\frac{1}{8} \to \cos\alpha\cos 2\alpha\cos 4\alpha=\frac{1}{8}$

$\frac{\sin\alpha\cos\alpha\cos 2\alpha\cos 4\alpha}{\sin\alpha}=\frac{1}{8} \to \frac{\frac{1}{8}\sin 8\alpha}{\sin\alpha}=\frac{1}{8}$

$\to \sin 8\alpha=\sin\alpha \to \begin{cases} 8\alpha=2k\pi+\alpha \to \alpha=\frac{2k\pi}{7} \\ 8\alpha=2k\pi+\pi-\alpha \to \alpha=\frac{2k\pi+\pi}{9} \end{cases}$

$\to \alpha=\{\frac{2\pi}{7},\frac{4\pi}{7},\frac{6\pi}{7},\frac{\pi}{9},\frac{\pi}{3},\frac{5\pi}{9},\frac{7\pi}{9}\} \to \alpha_{max}=\frac{7\pi}{9}$

البته ۰ و π غیرقابل قبول‌اند چون مخرج صفر

۱۰-

$\tan(3x)\tan(x)=1 \to \tan(3x)=\cot x=\tan(\frac{\pi}{2}-x)$

$\to 3x=k\pi+\frac{\pi}{2}-x \to 4x=k\pi-\frac{\pi}{2} \to x=\frac{k\pi}{4}-\frac{\pi}{8}=\frac{2k\pi-\pi}{8}$

$\to \pi<\frac{2k\pi-\pi}{8}<2\pi \to 8\pi<2k\pi-\pi<16\pi \to k=\{4,5,6,7\}$ یا ۵، ۶، ۷، ۸

$\to x=\{\frac{9\pi}{8},\frac{11\pi}{8},\frac{13\pi}{8},\frac{15\pi}{8}\}$

مجموع $=\frac{(9+11+13+15)\pi}{8}=\frac{48\pi}{8}=6\pi$