به نام خدا

۱۹ آبان

سامان نژادکرد — دوازدهم پسر B — تکلیف ۱۷

۱-

$8\cos x - \tan^2 x = 1 \Rightarrow 8\cos x = 1 + \tan^2 x = \frac{1}{\cos^2 x}$

$\Rightarrow 8\cos^3 x = 1 \Rightarrow \cos x = \frac{1}{2} \Rightarrow x = \begin{cases} 2k\pi + \frac{\pi}{3} \Rightarrow \frac{\pi}{3} \\ 2k\pi - \frac{\pi}{3} \Rightarrow \frac{5\pi}{3} \end{cases}$ در بازه $[0, 2\pi]$

۲

دو جواب دارد

$\{\frac{\pi}{3}, \frac{5\pi}{3}\}$

۲-

$5\sin^2 x + 2\cos 3x = -2 \Rightarrow 5(1-\cos^2 x) + 2(4\cos^3 x - 3\cos x) = -2$

$\Rightarrow 5 - 5\cos^2 x + 8\cos^3 x - 6\cos x = -2 \Rightarrow 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0$

$\Rightarrow (\cos x + 1)(8\cos^2 x - 13\cos x + 7) = 0$ ۲

$\Rightarrow \begin{cases} \cos x = -1 \Rightarrow \boxed{x = \pi} \text{ یا } \boxed{x = \cancel{-\pi}} \\ 8\cos^2 x - 13\cos x + 7 = 0 \Rightarrow \Delta < 0 \end{cases}$

۳-

$2\sin x\cos 2x + \sin x = 1 \Rightarrow 2\sin x(1 - 2\sin^2 x) + \sin x = 1$

$\Rightarrow 2\sin x - 4\sin^3 x + \sin x = 1 \Rightarrow 4\sin^3 x - 3\sin x + 1 = 0$ ۱,۵

$\Rightarrow (\sin x + 1)(4\sin^2 x - 4\sin x + 1) = 0$

$\Rightarrow \begin{cases} \sin x = -1 \Rightarrow x = 2k\pi - \frac{\pi}{2} \Rightarrow x = \frac{3\pi}{2} \\ 4\sin^2 x - 4\sin x + 1 = 0 \Rightarrow \sin x = +\frac{1}{2} \Rightarrow x = 2k\pi + \frac{\pi}{6} \Rightarrow x = \frac{\pi}{6} \\ \quad 2k\pi + \frac{5\pi}{6} \Rightarrow x = \frac{5\pi}{6} \end{cases}$

جمع جواب‌ها!

$x = \{\frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}\}$

$\frac{5\pi}{6} + \frac{\pi}{6} + \frac{3\pi}{2} = \frac{5\pi}{2}$

۴-

$\frac{1}{\sin(\frac{\pi}{2} + 2x)} + \frac{1}{\cos(\frac{\pi}{2} + 4x)} = \frac{1}{\cos 2x} + \frac{-1}{\sin 4x} = \frac{\sin 4x - \cos 2x}{\cos 2x \sin 4x} = 0$ ۲

$\Rightarrow 2\sin 2x\cos 2x - \cos 2x = 0 \Rightarrow \begin{cases} \cos 2x = 0 \quad \text{به خاطر تعریف غ ق ق} \\ \sin 2x = \frac{1}{2} \Rightarrow 2x = 2k\pi + \frac{\pi}{6} \Rightarrow x = \frac{\pi}{12} \\ \quad 2k\pi + \frac{5\pi}{6} \Rightarrow x = \frac{5\pi}{12} \end{cases}$

$\Rightarrow x_2 - x_1 = \frac{7\pi}{12} - \frac{\pi}{12} = \frac{6\pi}{12} = \frac{\pi}{2} = ۴$

$\Rightarrow \tan 2\alpha = \tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$

۸ -

$$\cos\left(x+\frac{\pi}{4}\right) = \frac{\cos x - \sin x}{\sqrt{2}} = \frac{1}{\sqrt{3}}$$

$\Rightarrow \boxed{\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}} \Rightarrow (\cos x - \sin x)^2 = 1 - \sin 2x = \frac{2}{3} \Rightarrow \sin 2x = \frac{1}{3}$

$\Rightarrow m(\cos x - \sin x) - 3\sqrt{6}\sin 2x = m\left(\frac{\sqrt{6}}{3}\right) - 3\sqrt{6}\sin 2x = \sqrt{6}$

$\Rightarrow m\left(\frac{\sqrt{6}}{3}\right) - \sqrt{6} = \sqrt{6} \Rightarrow \boxed{m = 6}$

۹ -

$$\sin\left(x+\frac{\pi}{6}\right)\underbrace{\cos\left(x-\frac{\pi}{3}\right)}_{\sin\left(\frac{\pi}{6}+x\right)} = 1 \Rightarrow \sin^2\left(\frac{\pi}{6}+x\right) = 1$$

$$\Rightarrow \begin{cases} \sin\left(x+\frac{\pi}{6}\right) = 1 \Rightarrow x+\frac{\pi}{6} = 2k\pi + \frac{\pi}{2} \Rightarrow x = 2k\pi + \frac{\pi}{3} \\ \sin\left(x+\frac{\pi}{6}\right) = -1 \Rightarrow x+\frac{\pi}{6} = 2k\pi - \frac{\pi}{2} \Rightarrow x = 2k\pi - \frac{2\pi}{3} \end{cases}$$

$\Rightarrow x = \frac{\pi}{3}, \ x = \frac{4\pi}{3} \Rightarrow$ ۲ جواب

۷ -

$\sin x + \sqrt{3}\cos x = \sqrt{2} \Rightarrow \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$

$$\Rightarrow \sin\left(x+\frac{\pi}{3}\right) = \frac{\sqrt{2}}{2} \Rightarrow \begin{cases} x+\frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \Rightarrow x = 2k\pi - \frac{\pi}{12} \\ x+\frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \Rightarrow x = 2k\pi + \frac{5\pi}{12} \end{cases}$$

$$\Rightarrow \begin{cases} x = \frac{24k\pi - \pi}{12} \Rightarrow k = 0, 1 \Rightarrow x = \left\{-\frac{\pi}{12}, \frac{23\pi}{12}\right\} \\ x = \frac{24k\pi + 5\pi}{12} \Rightarrow k = 0 \Rightarrow x = \left\{\frac{5\pi}{12}\right\} \end{cases}$$

$-\pi \le x \le 2\pi$

$\Rightarrow -\frac{\pi}{12} + \frac{5\pi}{12} + \frac{23\pi}{12} = \frac{27\pi}{12}$

۸ـ

$4\sin x \sin\left(\frac{3\pi}{2} - x\right) = 1$

$\rightarrow -4\sin x \cos x = 1 \rightarrow -2\sin 2x = 1 \rightarrow \sin 2x = -\frac{1}{2}$

$\rightarrow \begin{cases} 2x = 2k\pi + \frac{11\pi}{6} \rightarrow x = k\pi + \frac{11\pi}{12} \rightarrow x = \frac{11\pi}{12}, \frac{23\pi}{12} \\ 2x = 2k\pi + \frac{7\pi}{6} \rightarrow x = k\pi + \frac{7\pi}{12} \rightarrow x = \frac{7\pi}{12}, \frac{19\pi}{12} \end{cases}$

$\rightarrow$ مجموع: $\frac{11\pi}{12} + \frac{23\pi}{12} + \frac{7\pi}{12} + \frac{19\pi}{12} = \frac{60\pi}{12} = 5\pi$

۹ـ

$(1+\cos 2a)(1+\cos 4a)(1+\cos 8a) = \frac{1}{8}$

$\rightarrow 2\cos^2 a \times 2\cos^2 2a \times 2\cos^2 4a = \frac{1}{8} \rightarrow \cos a \cos 2a \cos 4a = \frac{1}{8}$

۱/۵

$\frac{\sin a \cos a \cos 2a \cos 4a}{\sin a} = \frac{1}{8} \rightarrow \frac{\frac{1}{8}\sin 8a}{\sin a} = \frac{1}{8}$

$\rightarrow \sin 8a = \sin a \rightarrow \begin{cases} 8a = 2k\pi + a \rightarrow a = \frac{2k\pi}{7} \\ 8a = 2k\pi + \pi - a \rightarrow a = \frac{2k\pi + \pi}{9} \end{cases}$

MAX $\rightarrow \frac{8\pi}{9}$

$\rightarrow a = \left\{\frac{2\pi}{7}, \frac{4\pi}{7}, \frac{6\pi}{7}\right.$ و $\left.\frac{\pi}{9}, \frac{\pi}{3}, \frac{5\pi}{9}, \frac{7\pi}{9}\right\} \rightarrow a_{max} = \frac{7\pi}{9}$

البته ۰ و π غیرقابل قبول [illegible]

۱۰ـ

$\tan(3x)\tan(x) = 1 \rightarrow \tan(3x) = \cot x = \tan\left(\frac{\pi}{2} - x\right)$

$\rightarrow 3x = k\pi + \frac{\pi}{2} - x \rightarrow 4x = k\pi - \frac{\pi}{2} \rightarrow x = \frac{k\pi}{4} - \frac{\pi}{8} = \frac{2k\pi - \pi}{8}$

$\rightarrow \pi < \frac{2k\pi - \pi}{8} < 2\pi \rightarrow 8\pi < 2k\pi - \pi < 16\pi \rightarrow k = \{5, 6, 7, 8\}$

$\rightarrow x = \left\{\frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}\right\}$

۲

مجموع $= \frac{(9+11+13+15)\pi}{8} = \frac{48\pi}{8} = 6\pi$

$$1+\cos 2\star = 2\cos^2\star$$

$$\left\{\begin{array}{l} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{array}\right\} \rightarrow 8\cos^2\alpha\,\cos^2 2\alpha\,\cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha\,\cos^2 2\alpha\,\cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\,\cos 2\alpha\,\cos 4\alpha = \frac{\pm 1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha} \star$$

$$\underbrace{(\sin\alpha\cos\alpha)}_{\sin 2\alpha}\cos 2\alpha\,\cos 4\alpha = \frac{\pm\sin\alpha}{8} \longrightarrow \frac{1}{2}\underbrace{(\sin 2\alpha\,\cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha}\cos 4\alpha = \frac{\pm\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha\,\cos 4\alpha)}_{\sin 8\alpha} = \frac{\pm\sin\alpha}{8} \longrightarrow \frac{1}{\cancel{8}}\sin 8\alpha = \frac{\pm\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \longrightarrow \left\{\begin{array}{l} 8\alpha = 2k\pi+\alpha \ \text{یا}\ 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi-\alpha \ \text{یا}\ 2k\pi+\pi+\alpha \end{array}\right.$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{\text{MAx } A} k=3 \rightarrow x = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{\text{MAx } A} k=4 \rightarrow x = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\text{MAx } A} k=2 \rightarrow x = \frac{5\pi}{7}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\text{MAx } A} k=3 \rightarrow x = \frac{7\pi}{9}$$

$$\rightarrow \boxed{\text{MAx}\,A = \frac{8\pi}{9}}$$

★ A نمی‌تواند برابر خود π باشد چون طبق ★ $\sin\alpha$ را مخالف صفر در نظر گرفته‌ایم!