۱- $8\cos x - \tan^2 x = 1 \to 8\cos x = \underbrace{1+\tan^2 x}_{\frac{1}{\cos^2 x}} \to 8\cos^3 x = 1 \to \cos^3 x = \frac{1}{8} \to \cos x = \frac{1}{2}$ ۲

$\to x = 2k\pi \pm \frac{\pi}{3} \to$ دو جواب در بازه‌ی $[0, 2\pi]$

۲- $5\sin^2(x) + 2\cos(3x) = -2$ ۰

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۳- $2\sin x\cos 2x + \sin x = 1$ ۲

$\to 2\sin x(1-2\sin^2 x) + \sin x = 1 \to -4\sin^3 x + 3\sin x - 1 = 0$

$\to (\sin x + 1)(-4\sin^2 x + 4\sin x - 1) = 0 \begin{cases} \sin x = -1 \to x = 2k\pi + \frac{3\pi}{2} \\ \sin x = \frac{1}{2} \Rightarrow x = 2k\pi + \frac{\pi}{6},\ 2k\pi + \frac{5\pi}{6} \end{cases} \Rightarrow \frac{\pi}{6} + \frac{5\pi}{6} + \frac{3\pi}{2} = \frac{5\pi}{2}$

۴- $\frac{1}{\underbrace{\sin\left(\frac{\pi+4x}{2}\right)}_{\cos 2x}} + \frac{1}{\underbrace{\cos\left(\frac{\pi+4x}{2}\right)}_{-\sin 2x}} = 0 \to \frac{\cos 2x - \sin 2x}{-\cos 2x\sin 2x} = 0 \to \cos 2x - \underbrace{\sin 4x}_{2\sin 2x\cos 2x} = 0 \to \cos 2x(1 - 2\sin 2x) = 0$

$2\alpha = \frac{2\pi}{3} \to \tan 2\alpha = -\sqrt{3}$ ۲

$\to \begin{cases} \cos 2x = 0 \quad \times \quad \text{مخرج صفر نباید باشد} \\ \sin 2x = \frac{1}{2} \to 2x = 2k\pi + \frac{\pi}{6},\ 2k\pi + \frac{5\pi}{6} \to x = k\pi + \frac{\pi}{12},\ k\pi + \frac{5\pi}{12} \end{cases} \to \frac{\pi}{12}, \frac{5\pi}{12} \to \alpha = \frac{4\pi}{12} = \frac{\pi}{3}$

۵- $\cos\left(x+\frac{\pi}{4}\right) = \frac{1}{\sqrt{3}} \to \cos\left(x+\frac{\pi}{4}\right) = \sin\left(-x+\frac{\pi}{4}\right) = -\sin\left(x-\frac{\pi}{4}\right) \to -\sin\left(x-\frac{\pi}{4}\right) = \frac{-(\sin x - \cos x)}{\sqrt{2}}$

$\to \frac{\cos x - \sin x}{\sqrt{2}} = \frac{1}{\sqrt{3}} \to \boxed{\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}} \to \frac{6}{9} = \underbrace{\cos^2 + \sin^2}_{1} \underbrace{- 2\sin x\cos x}_{-\sin 2x} \to \boxed{\sin 2x = \frac{1}{3}}$ ۲

$m(\cos x - \sin x) - 3\sqrt{6}\sin(2x) = \sqrt{6} \to \frac{m\sqrt{6}}{3} - \frac{3\sqrt{6}}{3} = \sqrt{6} \to \frac{m\sqrt{6}}{3} = 2\sqrt{6} \to m = 6$

۶- $\sin\left(x+\frac{\pi}{6}\right)\underbrace{\cos\left(x-\frac{\pi}{3}\right)}_{\cos\alpha = \cos -\alpha} = 1 \to \sin\left(x+\frac{\pi}{6}\right)\underbrace{\cos\left(\frac{\pi}{3}-x\right)}_{\sin\left(x+\frac{\pi}{6}\right)} = 1 \to \sin^2\left(x+\frac{\pi}{6}\right) = 1 \to \sin\left(x+\frac{\pi}{6}\right) = \pm 1$

$\to x + \frac{\pi}{6} = k\pi + \frac{\pi}{2} \to x = k\pi + \frac{\pi}{3}$

۲ جواب در بازه‌ی $[0, 2\pi]$ ۲

$\sin x+\sqrt{3}\cos x=\sqrt{2} \to \sqrt{1+(\sqrt{3})^2}=2 \;\; \sin(x+\alpha) \to 2\sin(x+\frac{\pi}{3})=\sqrt{2} \to \sin(x+\frac{\pi}{3})=\frac{\sqrt{2}}{2}$ ۷-

$\tan\alpha=\sqrt{3} \to \alpha=\frac{\pi}{3}$

$\to x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4} \;,\; 2k\pi+\frac{3\pi}{4}$ ۱/۵

$\to x=2k\pi-\frac{\pi}{12} \;,\; 2k\pi+\frac{5\pi}{12}$ → ۳ جواب در بازه‌ی $[0,2\pi]$

مجموع جواب ها؟

$\frac{5\pi}{12}+\frac{23\pi}{12}-\frac{\pi}{12}=\frac{27\pi}{12}=\frac{9\pi}{4}$

$4\sin x\sin(\frac{3\pi}{2}-x)=1 \to -4\sin x\cos x=1 \to -2\sin 2x=1 \to \sin 2x=-\frac{1}{2}$ ۱/۵ ۸-

$\to 2x=2k\pi+\frac{7\pi}{6} \;,\; 2k\pi+\frac{11\pi}{6} \to x=k\pi+\frac{7\pi}{12} \;,\; k\pi+\frac{11\pi}{12}$ → ۴ جواب در بازه‌ی $[0,2\pi]$

جمع جواب ها؟

$\frac{7\pi}{12}+\frac{19\pi}{12}+\frac{11\pi}{12}+\frac{23\pi}{12}=\frac{60\pi}{12}=5\pi$

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* A نمی‌تواند برابر خود π باشد چون طبق * sinα را مخالف صفر در نظر گرفته ایم!

$(1+\cos(2\alpha))(1+\cos(4\alpha))(1+\cos(8\alpha))=\frac{1}{8} \xrightarrow{1+\cos 2x=2\cos^2 x} (2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha)=\frac{1}{8}$ ۹-

$\to 8[(\cos\alpha)(\cos 2\alpha)(\cos 4\alpha)]^2=\frac{1}{8} \to (\cos\alpha)(\cos 2\alpha)(\cos 4\alpha)=\pm\frac{1}{8} \xrightarrow{\times\sin\alpha} \frac{\sin 8\alpha}{8}=\pm\frac{1}{8}\sin\alpha$

$\sin 8\alpha=\sin\alpha \to \alpha=\frac{2}{7}k\pi \;,\; \alpha=\frac{2}{9}k\pi+\frac{\pi}{9}$

$\sin 8\alpha=-\sin\alpha=\sin(-\alpha) \to \alpha=\frac{2k\pi}{9} \;,\; \alpha=\frac{2}{7}k\pi+\frac{\pi}{7}$ → Max $=2\pi$

در بازه‌ی $[0,\pi]$ ۱/۵

$\tan(3x)\tan(x)=1 \to \tan 3x=\cot x \to 3x=k\pi+\frac{\pi}{2}-x \to 4x=k\pi+\frac{\pi}{2} \to x=\frac{k\pi}{4}+\frac{\pi}{8}$ ۱۰

$\to \frac{5\pi}{8},\frac{7\pi}{8} \to$ مجموع $=3\pi$ ۱/۵

کامل بگی

$\tan 3x=\frac{1}{\tan x}=\cot x \to \tan 3x=\tan(\frac{\pi}{2}-x)$ ۱۰

$3x=k\pi+\frac{\pi}{2}-x \to 4x=k\pi+\frac{\pi}{2} \to x=\frac{k\pi}{4}+\frac{\pi}{8}$

k	۴	۵	۶	۷
x	$\frac{9\pi}{8}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{15\pi}{8}$

$S=\frac{(9+11+13+15)\pi}{8}=\frac{48\pi}{8}=\boxed{6\pi}$

- ۳

$$\cos 3x = 2\cos^2\frac{3x}{2} - 1$$ (فرمول طلایی)

$$5\sin^2 x + 2\left(2\cos^2\frac{3x}{2} - 1\right) = -2 \rightarrow 5\sin^2 x + 4\cos^2\frac{3x}{2} \cancel{-2} = \cancel{-2}$$

حاصل جمع دو عدد نامنفی صفر است پس هر دو صفر بوده اند!

$$5\sin^2 x = 0 \rightarrow \sin x = 0 \rightarrow x = k\pi \xrightarrow{-\pi \le x \le \pi} x = \pi$$ یا $-\pi$ یا 0

$$4\cos^2\frac{3x}{2} = 0 \rightarrow \cos\frac{3x}{2} = 0 \rightarrow \frac{3x}{2} = k\pi + \frac{\pi}{2} \rightarrow x = \frac{2k\pi + \pi}{3}$$

$$\xrightarrow{-\pi \le x \le \pi} x = -\pi$$ یا $-\frac{\pi}{3}$ یا $\frac{\pi}{3}$ یا π

اشتراک دو مجموعه جواب $\{-\pi, \pi\}$ است

۴

$$\cos\left(x - \frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3} - x\right)$$

$$\frac{\pi}{3} - x + x + \frac{\pi}{6} = \frac{\pi}{2} \longrightarrow \cos\left(\frac{\pi}{3} - x\right) = \sin\left(x + \frac{\pi}{6}\right)$$

$$\sin^2\left(x + \frac{\pi}{6}\right) = 1 \rightarrow \sin\left(x + \frac{\pi}{6}\right) = \pm 1 \rightarrow x + \frac{\pi}{6} = k\pi + \frac{\pi}{2}$$

$$x = k\pi + \frac{\pi}{3} \xrightarrow{0 \le x \le 2\pi} k = 0 \rightarrow x = \frac{\pi}{3}, \quad k = 1, \ x = \frac{4\pi}{3}$$

معادله ۲ جواب در بازه دارد

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۷ عبارت را در $\frac{1}{2}$ ضرب می‌کنیم.

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

$$\cos 60^\circ \sin x + \sin 60^\circ \cos x = \frac{\sqrt{2}}{2} \longrightarrow \sin\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

$$\begin{cases} x + \frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \rightarrow x = 2k\pi - \frac{\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k = 0, 1 \rightarrow x = -\frac{\pi}{12}, \frac{23\pi}{12} \\ x + \frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \rightarrow x = 2k\pi + \frac{5\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k = 0 \rightarrow x = \frac{5\pi}{12} \end{cases}$$

$$S = \frac{23\pi}{12} - \frac{\pi}{12} + \frac{5\pi}{12} = \frac{27\pi}{12} = \boxed{\frac{9\pi}{4}}$$

$1+\cos 2\star = 2\cos^2\star$

$$\left\{\begin{array}{l} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{array}\right\} ? \rightarrow 8\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha \cos 2\alpha \cos 4\alpha = \frac{\pm 1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha \ \star}$$

$$\underbrace{(\sin\alpha \cos\alpha)}_{\sin 2\alpha} \cos 2\alpha \cos 4\alpha = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{2}\underbrace{(\sin 2\alpha \cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha} \cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha \cos 4\alpha)}_{\sin 8\alpha} = \frac{\pm\sin\alpha}{8} \rightarrow \frac{1}{\cancel{8}}\sin 8\alpha = \pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \rightarrow \left\{\begin{array}{l} 8\alpha = 2k\pi+\alpha \ \text{یا}\ 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi-\alpha \ \text{یا}\ 2k\pi+\pi+\alpha \end{array}\right.$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{\text{MAX } A} k=3 \rightarrow x = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{\text{MAX } A} k=4 \rightarrow x = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\text{MAX } A} k=2 \rightarrow x = \frac{5\pi}{7} \qquad \rightarrow \boxed{\text{MAX } A = \frac{8\pi}{9}}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\text{MAX } A} k=3 \rightarrow x = \frac{7\pi}{9}$$

★ A نمی‌تواند برابر خود π باشد چون طبق ★ sinα را مخالف صفر در نظر گرفته‌ایم!