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پاسخنامه تشریحی تکلیف شماره .... کلاس ....

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الف) $\cos 2\alpha = 3\sin\alpha - 1 \longrightarrow \cos^2\alpha - \sin^2\alpha = 3\sin\alpha - 1 \longrightarrow 1-\sin^2\alpha - \sin^2\alpha = 3\sin\alpha - 1$

$\longrightarrow 2\sin^2\alpha + 3\sin\alpha - 2 = 0$ ، $\sin\alpha = -2 \rightarrow$ غ ق ق ، $\boxed{\sin\alpha = \frac{1}{2}} \longrightarrow \boxed{2k\pi + \frac{\pi}{6} \ , \ 2k\pi + \frac{5\pi}{6}}$

ب) $\cos 2\alpha + \cos\alpha - 2 = 0 \longrightarrow \cos^2\alpha - \sin^2\alpha + \cos\alpha - 2 = 0 \longrightarrow \cos^2\alpha + \cos^2\alpha - 1 + \cos\alpha - 2 = 0$

$2\cos^2\alpha + \cos\alpha - 3 = 0$ ، $\cos\alpha = 1 \longrightarrow \boxed{2k\pi}$ ، $\cos\alpha = -\frac{3}{2} \rightarrow$ غ ق ق

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الف) $\sin^4\alpha - \cos^4\alpha = \sin 4\alpha \longrightarrow -\cos 2\alpha = 2\sin 2\alpha \cancel{\cos 2\alpha} \longrightarrow -1 = 2\sin 2\alpha \rightarrow \sin 2\alpha = -\frac{1}{2}$

$2\alpha = 2k\pi + (-\frac{\pi}{6}) \longrightarrow \boxed{\alpha = k\pi - \frac{\pi}{12}}$

$2\alpha = 2k\pi + \pi - (-\frac{\pi}{6}) \longrightarrow 2\alpha = 2k\pi + \frac{7\pi}{6} \longrightarrow \boxed{\alpha = k\pi + \frac{7\pi}{12}}$

ب) $2\cos^2\alpha + 2\sin\alpha\cos\alpha = 1 \longrightarrow 2\sin\alpha\cos\alpha = 1 - 2\cos^2\alpha \longrightarrow \sin 2\alpha = -\cos 2\alpha$

$\xrightarrow{\div \cos 2\alpha} \tan 2\alpha = -1 \longrightarrow 2\alpha = k\pi - \frac{\pi}{4} \longrightarrow \boxed{\alpha = \frac{k\pi}{2} - \frac{\pi}{8}}$

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الف) $\cot\alpha(2\cos\alpha + 1) = \frac{1}{\sin\alpha}$ ، $\frac{\cos\alpha(2\cos\alpha+1)}{\sin\alpha} = \frac{1}{\sin\alpha} \longrightarrow 2\cos^2\alpha + \cos\alpha - 1 = 0$

$\longrightarrow \cos\alpha = \frac{1}{2} \longrightarrow \boxed{2k\pi \pm \frac{\pi}{3}}$

$\cos\alpha = -1 \longrightarrow \boxed{2k\pi + \pi}$

اگر $\cos x = -1$ باشد، $\sin x = 0$ است و عبارت تعریف نشده می‌شود، پس جواب غیر قابل قبول محسوب می‌شود

ب) $2\sin^2\alpha - \sin\alpha\cos\alpha + \cos^2\alpha = 2 \longrightarrow \sin^2\alpha - \sin\alpha\cos\alpha = 1 \rightarrow \frac{1-\cos 2\alpha}{2} - \frac{\sin 2\alpha}{2}$

$1 - \cos 2\alpha - \sin 2\alpha = 2 \longrightarrow \sin 2\alpha + \cos 2\alpha = -1 \longrightarrow \sqrt{2}\sin(2\alpha + \frac{\pi}{4}) = -1$

$\longrightarrow \sin(2\alpha + \frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$

$2\alpha + \frac{\pi}{4} = 2k\pi - \frac{\pi}{4} \longrightarrow 2\alpha = 2k\pi - \frac{\pi}{2} \rightarrow \boxed{\alpha = k\pi - \frac{\pi}{4}}$

$2\alpha + \frac{\pi}{4} = 2k\pi + \pi - (-\frac{\pi}{4})$ ، $2\alpha = 2k\pi + \pi \rightarrow \boxed{\alpha = k\pi + \frac{\pi}{2}}$

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الف) $\cos(\frac{\pi}{4} + x)\cos(x - \frac{\pi}{4}) = \frac{1}{4} \longrightarrow \frac{\sqrt{2}}{2}(\cos x - \sin x) \times \frac{\sqrt{2}}{2}(\cos x + \sin x) = \cos^2 x - \sin^2 x = \frac{1}{2}$

$\longrightarrow \cos 2\alpha = \frac{1}{2} \longrightarrow 2\alpha = 2k\pi \pm \frac{\pi}{3} \longrightarrow \boxed{\alpha = k\pi \pm \frac{\pi}{6}}$

ب) $\cos(2x - \frac{\pi}{9}) = -\sin(2x) \longrightarrow \cos(2x - \frac{\pi}{9}) = \sin(-2x) \longrightarrow \cos(2x - \frac{\pi}{9}) = \cos(\frac{\pi}{2} + 2x)$

$\longrightarrow 2x - \frac{\pi}{9} = 2k\pi \pm (2x + \frac{\pi}{2}) \longrightarrow 2x - \frac{\pi}{9} = k\pi - 2x - \frac{\pi}{2} \longrightarrow 4x = 2k\pi - \frac{7\pi}{18}$

$\boxed{x = \frac{k\pi}{2} - \frac{7\pi}{72}}$

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$\frac{\sin 6x + \sin 2x}{\sin 4x} - \sin^2 x = \cos^2 x \longrightarrow \frac{2\sin 4x\cos 2x + \sin 4x}{\sin 4x}$

$\longrightarrow \frac{\sin 4x(2\cos 2x + 1)}{\sin 4x} - \sin^2 x = \cos^2 x \longrightarrow 2\cos 2x + 1 = 1 \rightarrow 2\cos 2x = 0$

$\cos 2x = 0 \longrightarrow 2x = k\pi + \frac{\pi}{2} \longrightarrow \boxed{x = \frac{k\pi}{2} + \frac{\pi}{4}}$

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$\frac{x}{2}=\alpha \longrightarrow \frac{\sin\alpha}{1+\cos\alpha}=\frac{1-\cos\alpha}{\sin\alpha} \longrightarrow \underbrace{\sin^2\alpha}_{1-\cos^2\alpha}=1+\cos^2\alpha+2\cos\alpha$ ۶

$\longrightarrow 2\cos^2\alpha+2\cos\alpha=0 \longrightarrow 2\cos\alpha(\cos\alpha+1)=0 \longrightarrow \cos\alpha=0 \ ,\ \cos\alpha=-1$

$\longrightarrow \cos\frac{x}{2}=0 \longrightarrow \frac{x}{2}=k\pi+\frac{\pi}{2} \longrightarrow x=2k\pi+\pi \longrightarrow$ $k=-1 \rightarrow x=-\pi$, $k=0 \rightarrow x=\pi$

$\cos\frac{x}{2}=-1 \longrightarrow \frac{x}{2}=2k\pi+\pi \longrightarrow x=4k\pi+2\pi \longrightarrow$ غیرقابل قبول

جواب های ما: $(-\pi,\frac{3\pi}{2})$

$x=\pi \ ,\ x=-\pi \longrightarrow |\pi-(-\pi)|=2\pi$ ✓

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$\cos^2x-\sin^2x\cos3x=1 \longrightarrow -\sin^2x\cos3x=1-\cos^2x \longrightarrow -\sin^2x\cos3x=\sin^2x$ ۷

$\cos3x=-1 \longrightarrow 3x=2k\pi+\pi \longrightarrow x=\frac{2k\pi}{3}+\frac{\pi}{3} \longrightarrow$ ۳ جواب دارد

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$\frac{1-\tan x}{1+\tan x}=\tan3x \longrightarrow \frac{\frac{\cos x-\sin x}{\cos x}}{\frac{\cos x+\sin x}{\cos x}}=\frac{\cos x-\sin x}{\cos x+\sin x}=\frac{\sin3x}{\cos3x}$ ۸

$\cos3x\cos x-\cos3x\sin x=\sin3x\cos x+\sin3x\sin x \longrightarrow \cos3x\cos x-\sin3x\sin x=\sin3x\cos x+\cos3x\sin x$

$\cos(4x)=\sin(4x) \xrightarrow{\div\cos4x} 1=\tan4x \longrightarrow 4x=k\pi+\frac{\pi}{4} \longrightarrow \boxed{x=\frac{k\pi}{4}+\frac{\pi}{16}}$ ✓

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$\sqrt{1+\sin2\alpha}=\sqrt{2}\cos2\alpha \longrightarrow 1+\sin2\alpha=2\cos^2 2\alpha$ ۹

$\longrightarrow \sin2\alpha=2\cos^2 2\alpha-1 \longrightarrow \sin2\alpha=\cos^2 2\alpha-\sin^2 2\alpha \longrightarrow$

جواب ها ؟ ۱٫۵

$\sin2\alpha=\cos4\alpha \longrightarrow 2\alpha=2k\pi+\frac{\pi}{2}-4\alpha \longrightarrow 6\alpha=2k\pi+\frac{\pi}{2}$

$2\alpha=2k\pi-\frac{\pi}{2}+4\alpha \longrightarrow 2\alpha=\frac{\pi}{2}-2k\pi$

$\alpha=\frac{\pi}{4}-k\pi$

$\boxed{\alpha=\frac{k\pi}{3}+\frac{\pi}{12} \quad \alpha=\frac{\pi}{4}-k\pi}$ ✓

الف) $\sin^4x-\cos^4x=1 \longrightarrow -\cos2\alpha=1 \longrightarrow \cos2\alpha=-1$ ۱۰

$2\alpha=2k\pi+\pi \longrightarrow \boxed{\alpha=k\pi+\frac{\pi}{2}}$ ✓

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ب) $\sin^3x-\cos^3x=1$

$\sin x=1 \longrightarrow \cos x=0 \longrightarrow$ ✗

$\sin x=0 \longrightarrow \cos x=-1 \longrightarrow$ ✓

$\sin x=-1 \longrightarrow \cos x=0 \longrightarrow$ ✓

$\sin x=0 \longrightarrow \cos x=1 \longrightarrow$ ✗

با رسم دایره

$\boxed{2k\pi \ ,\ 2k\pi+\frac{3\pi}{2}}$

سوال ۷

$$\cos^2 x - \sin^2 x \cos 3x = 1 \xrightarrow{\cos^2 x = 1-\sin^2 x}$$

$$\cancel{1} - \sin^2 x - \sin^2 x \cos 3x = \cancel{1} \longrightarrow \sin^2 x (1 + \cos 3x) = 0 \begin{cases} \sin x = 0 \\ \cos 3x = -1 \end{cases}$$

$$\sin x = 0 \longrightarrow x = k\pi \longrightarrow x = 0, \pi, 2\pi$$

$$\cos 3x = -1 \longrightarrow 3x = 2k\pi + \pi \longrightarrow x = \frac{2k\pi + \pi}{3} \longrightarrow x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

معادله ۵ جواب دارد

سوال ۹

$$\sqrt{1 + \sin 2x} = \sqrt{2} \cos 2x \xrightarrow{\text{توان ۲}} 1 + \sin 2x = 2\cos^2 2x$$

$$1 + \sin 2x = 2 - 2\sin^2 2x \longrightarrow 2\sin^2 2x + \sin 2x - 1 = 0 \begin{cases} \sin 2x = -1 \\ \sin 2x = +\frac{1}{2} \end{cases}$$

$$\sin 2x = -1 \rightarrow 2x = 2k\pi - \frac{\pi}{2} \xrightarrow[k=1]{0 \le x \le \pi} x = \frac{3\pi}{4} \checkmark$$

$$\sin 2x = \frac{1}{2} \rightarrow 2x = 2k\pi + \frac{\pi}{6} \text{ یا } 2k\pi + \frac{5\pi}{6} \xrightarrow[k=0]{0 \le x \le \pi} x = \frac{\pi}{12} \checkmark \text{ یا } \frac{5\pi}{12} \times$$

چون عبارت به توان ۲ رسیده است جواب زائد تولید می‌کند

به ازای $x = \frac{5\pi}{12}$ ، $\cos 2x$ منفی می‌شود، چون جواب رادیکال است غ ق ق است

* معادله ۲ جواب دارد