

$$2\cos^2 \alpha - 1 + \cos \alpha - 2 = 0$$

$$\rightarrow 2t^2 + t - 3 = 0 \rightarrow t = 1 \text{ و } -\frac{3}{2}$$

$$\rightarrow \cos \alpha = 1 \rightarrow \alpha = 2k\pi$$

$$(ب) \quad 1 - 2\sin^2 \alpha = 3\sin \alpha - 1 \quad (الف)$$

$$\rightarrow 2t^2 + 3t - 2 = 0 \rightarrow t = \frac{1}{2} \text{ و } -2$$

$$\rightarrow \sin \alpha = \frac{1}{2} \rightarrow \alpha = (2k\pi + \frac{\pi}{6}) \cup (2k\pi + \frac{5\pi}{6})$$

$$2\cos^2 \alpha - 1 + 2\sin \alpha \cos \alpha = 0$$

$$\rightarrow \cos^2 \alpha + \sin^2 \alpha = 0 \rightarrow \cos^2 \alpha = \cos(\frac{\pi}{2} + \alpha) = 0$$

$$\rightarrow \cos^2 \alpha = + \cos(\frac{\pi}{2} - \alpha) \rightarrow + \alpha = 2k\pi \pm \frac{\pi}{2} - \alpha$$

$$\rightarrow \alpha = 2k\pi - \frac{\pi}{2} \rightarrow \frac{k\pi}{2} - \frac{\pi}{2} = \alpha$$

$$(ب) \quad \sin^2 \alpha - \cos^2 \alpha = \sin^2 \alpha \quad (الف)$$

$$-\cos^2 \alpha = 2\sin \alpha \cos \alpha$$

$$\rightarrow \cos^2 \alpha = 0 \rightarrow \sin \alpha = -\frac{1}{2}$$

$$\rightarrow \alpha = k\pi + \frac{7\pi}{6} \rightarrow \alpha = \frac{k\pi}{2} + \frac{7\pi}{6}$$

$$\rightarrow \alpha = 2k\pi - \frac{\pi}{6} \rightarrow \alpha = k\pi - \frac{\pi}{6}$$

$$\rightarrow \alpha = 2k\pi - \frac{5\pi}{6} \rightarrow \alpha = k\pi - \frac{5\pi}{6}$$

$$\rightarrow \sin^2 \alpha \rightarrow 2 - \cot \alpha + \cot^2 \alpha = 2(\cot^2 \alpha + 1) \quad (ب)$$

$$\rightarrow \cot \alpha = t \rightarrow t^2 - t + 1 = 2(t^2 + 1) \rightarrow t^2 + t = 0$$

$$\rightarrow t(t+1) = 0 \rightarrow \cot \alpha = 0 \rightarrow 1 \rightarrow \alpha = k\pi + \frac{\pi}{2}$$

$$\alpha = k\pi + \frac{\pi}{2}$$

$$\sin \alpha \neq 0$$

$$\rightarrow \cos \alpha (2\cos \alpha + 1) = 1$$

$$2\cos^2 \alpha + \cos \alpha - 1 = 0$$

$$\rightarrow \cos \alpha = -1, \cos \alpha = \frac{1}{2}$$

$$\rightarrow \alpha = 2k\pi, 2k\pi \pm \frac{\pi}{3}$$

$$\cos(\frac{\pi}{2} + \alpha - \frac{11\pi}{12}) = -\sin \alpha$$

$$\rightarrow \sin(\alpha - \frac{11\pi}{12}) = -\sin \alpha \rightarrow \alpha = \frac{k\pi}{2} + \frac{\pi}{12}$$

$$\rightarrow \alpha = \frac{11\pi}{12} = 2k\pi + \alpha$$

$$\rightarrow \alpha - \frac{11\pi}{12} = 2k\pi + \alpha \rightarrow \alpha = 2k\pi + \frac{\pi}{12}$$

$$(ب) \quad \cos(\frac{\pi}{2} + \alpha - \frac{\pi}{2}) \cos(\alpha - \frac{\pi}{2}) = \frac{1}{2} \quad (الف)$$

$$\rightarrow -\sin(t) \cos(t) = \frac{1}{2}$$

$$\rightarrow -\sin(2t) = \frac{1}{2} \rightarrow 2t = \frac{7\pi}{6} \rightarrow \alpha = 2k\pi + \frac{7\pi}{6}$$

$$\rightarrow \alpha = -k\pi + \frac{\pi}{4}$$

$$\rightarrow \alpha = -k\pi - \frac{3\pi}{4}$$

$$\frac{\sin^2 \alpha (2\cos^2 \alpha + 1)}{\sin^2 \alpha} = 1 \rightarrow \cos^2 \alpha = 0 \rightarrow \alpha = 2k\pi \pm \frac{\pi}{2} \rightarrow \alpha = k\pi \pm \frac{\pi}{2}$$

$$\frac{\sin \frac{\alpha}{2}}{1 + \cos \frac{\alpha}{2}} = \frac{1 + \cos \frac{\alpha}{2}}{\sin \frac{\alpha}{2}} \xrightarrow{\text{! ملاحظه}} \sin \frac{\alpha}{2} = 1 + 2 \cos \frac{\alpha}{2} + \cos \frac{\alpha}{2}$$

$$\rightarrow 1 + 2 \cos \frac{\alpha}{2} + \cos \frac{\alpha}{2} + \cos \frac{\alpha}{2} - 1 = 2 \cos \frac{\alpha}{2} (\cos \frac{\alpha}{2} + 1) \Rightarrow \cos \frac{\alpha}{2} = 0$$

(با توجه به اینکه $\cos \frac{\alpha}{2} = 0$ در $\frac{\pi}{2}$ و $\frac{3\pi}{2}$ می باشد)

$$\rightarrow \frac{\alpha}{2} = k\pi + \frac{\pi}{2} \rightarrow \alpha = 2k\pi + \pi$$


(با توجه به اینکه $|- \pi - \pi| = 2\pi$)

$$\cos^2 \alpha - \sin^2 \alpha \cos^2 \alpha = \cos^2 \alpha + \sin^2 \alpha \rightarrow \sin^2 \alpha (1 + \cos^2 \alpha) = 0$$

$$\rightarrow \sin^2 \alpha = 0 \rightarrow \alpha = k\pi$$

(با توجه به اینکه $\cos^2 \alpha = -1 \rightarrow 2\alpha = 2k\pi + \pi$)

$$\rightarrow \frac{2k\pi}{4} + \frac{\pi}{2} = \alpha$$

$$\left\{ 0, \pi, 2\pi, \frac{\pi}{2}, \frac{3\pi}{2} \right\}$$

$$\frac{\tan \frac{\pi}{2} - \tan \alpha}{1 + \tan \frac{\pi}{2} \tan \alpha} = \tan \left(\frac{\pi}{2} - \alpha \right) = \tan \pi \rightarrow \pi = k\pi + \frac{\pi}{2} - \alpha$$

$$\rightarrow \pi = k\pi + \frac{\pi}{2} \rightarrow \alpha = \frac{k\pi}{2} + \frac{\pi}{4}$$

$$|\sin \alpha + \cos \alpha| = \sqrt{2} \cos \frac{\pi}{4} \rightarrow \sqrt{2} \left| \sin \left(\alpha + \frac{\pi}{4} \right) \right| = \sqrt{2} \cos \frac{\pi}{4}$$

$$|\cos \left(\frac{\pi}{2} - \alpha \right)| = \cos \frac{\pi}{4} \rightarrow \pi = 2k\pi + \frac{\pi}{2} + \alpha \rightarrow \pi = 2k\pi + \frac{\pi}{2}$$

$$\alpha = \frac{\pi}{4} \text{ و } \frac{3\pi}{4}$$

(با توجه به اینکه $\sin \alpha = \frac{1}{\sqrt{2}}$ و $\cos \alpha = \frac{1}{\sqrt{2}}$)

$$\begin{aligned} \pi &= 2k\pi + \frac{\pi}{2} + \alpha \rightarrow \alpha = 2k\pi + \frac{\pi}{2} \\ \pi &= 2k\pi + \frac{\pi}{2} - \alpha \rightarrow \alpha = 2k\pi - \frac{\pi}{2} \\ \pi &= 2k\pi + \frac{\pi}{2} + \alpha \rightarrow \alpha = 2k\pi + \frac{\pi}{2} \\ \pi &= 2k\pi + \frac{\pi}{2} - \alpha \rightarrow \alpha = 2k\pi - \frac{\pi}{2} \end{aligned}$$

(با توجه به اینکه $\sin \alpha = \frac{1}{\sqrt{2}}$ و $\cos \alpha = \frac{1}{\sqrt{2}}$)

$$(b) \text{ در این تیپ سوالات ۴ نکته اصلی را استادم می گوید}$$

$$\begin{aligned} 0^\circ &\rightarrow \checkmark \\ 90^\circ &\rightarrow \times \\ 180^\circ &\rightarrow \times \\ 270^\circ &\rightarrow \checkmark \end{aligned}$$

$$\alpha = (2k\pi) \cup (2k\pi - \frac{\pi}{2})$$

$$\alpha \in \left\{ 0, \frac{3\pi}{2}, 2\pi \right\}$$

$$(c) (\sin^2 + \cos^2)(\sin^2 - \cos^2) = 1$$

$$\rightarrow -\cos^2 \alpha = 1 \rightarrow \cos^2 \alpha = -1$$

$$\rightarrow \pi = 2k\pi + \pi \rightarrow \alpha = k\pi + \frac{\pi}{2}$$