امیر کاشف

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الف) $\cos 2x = 3\sin x - 1 \Rightarrow \cos^2 x - \sin^2 x = 3\sin x - 1$

$1 - 2\sin^2 x = 3\sin x - 1 \Rightarrow 2t^2 + 3t - 2 = 0 \rightarrow t = \frac{1}{2}$ ، $t = -2$ غ ق ق

$\sin x = t$

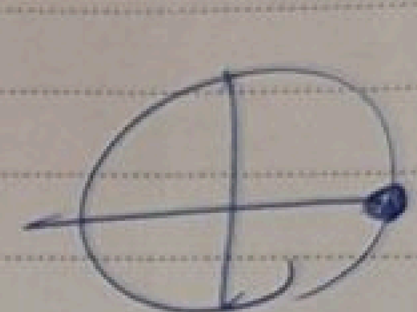
$\Rightarrow \sin x = 2k\pi + \frac{\pi}{6}$

$2k\pi + \pi - \frac{\pi}{6}$

ب) $\cos 2x + \cos x - 2 = 0 \Rightarrow 2\cos^2 x + \cos x - 3 = 0$

$\cos x = t \Rightarrow 2t^2 + t - 3 = 0 \quad \frac{-1 \pm \sqrt{1 + 24}}{4} \Rightarrow t = -\frac{6}{4}$ ✗

$t = 1$



$\cos x = \cos 2k\pi \Rightarrow x = 2k\pi$

۲

الف) $\sin^4 x - \cos^4 x = \sin 4x$

$\Rightarrow \underbrace{(\sin^2 x - \cos^2 x)}_{-\cos 2x}\underbrace{(\sin^2 x + \cos^2 x)}_{1} = \sin 4x$

$\Rightarrow -\cos 2x = 2\sin 2x \cos 2x$

$\cos 2x = 0 \Rightarrow 2x = k\pi + \frac{\pi}{2} \Rightarrow \boxed{x = \frac{k\pi}{2} + \frac{\pi}{4}}$

$\sin 2x = -\frac{1}{2} \Rightarrow 2x = 2k\pi - \frac{\pi}{6} \Rightarrow \boxed{x = k\pi - \frac{\pi}{12}}$

$2x = 2k\pi + \frac{7\pi}{6} \Rightarrow \boxed{x = k\pi + \frac{7\pi}{12}}$

ب) $\frac{2\cos^2 x + 2\sin x \cos x}{\sin 2x} = 1 \Rightarrow \underbrace{2\cos^2 x - 1}_{\cos 2x} = -\sin 2x$

$\Rightarrow \sin\left(\frac{\pi}{2} - 2x\right) = \sin(-2x)$

$\Rightarrow -2x = 2k\pi + \frac{\pi}{2} - 2x \Rightarrow$ ✗

$-2x = 2k\pi + \underbrace{\pi - \frac{\pi}{4}}_{\frac{5\pi}{4}} + 2x \Rightarrow -4x = 2k\pi + \frac{5\pi}{4} \Rightarrow \boxed{x = -\frac{k\pi}{2} - \frac{5\pi}{24}}$

۳

الف) $\cot x(2\cos x + 1) = \frac{1}{\sin x} \Rightarrow \frac{\cos x}{\sin x}(2\cos x + 1) = \frac{1}{\sin x} \Rightarrow 2\cos^2 x + \cos x = 1$

$2t^2 + t - 1 = 0$

$t = -1 \Rightarrow \cos x = -1 \Rightarrow \boxed{x = 2k\pi + \pi}$

$t = \frac{1}{2} \Rightarrow \cos x = \frac{1}{2} \Rightarrow \boxed{x = 2k\pi \pm \frac{\pi}{3}}$

ب) $2\sin^2 x - \sin x\cos x + \cos^2 x = 2 \quad \div \cos^2 x \Rightarrow 2\tan^2 x - \tan x + 1 = \frac{2}{\cos^2 x}$ $\quad \frac{1}{\cos^2 x} = 1+\tan^2 x$

$2\tan^2 x - \tan x + 1 = 2 + \cancel{2\tan^2 x} \Rightarrow \tan x = -1$

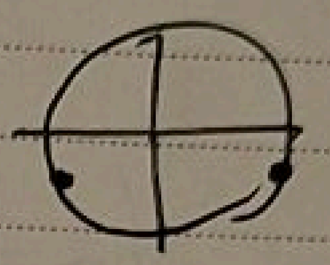
الف) $\cos(\frac{\pi}{4}+x)\cos(x-\frac{\pi}{4}) = \frac{1}{4}$ ۴

$\cos(\frac{\pi}{2} - \frac{\pi}{4})$

$-\sin(x-\frac{\pi}{4})$

$\Rightarrow \frac{\sin(2x-\frac{\pi}{2})}{-2} = \frac{1}{4} \Rightarrow$

$\sin(2x - \frac{\pi}{2}) = -\frac{1}{2}$



$\Rightarrow 2x - \frac{\pi}{2} = 2k\pi - \frac{\pi}{6}$

$\Rightarrow \boxed{x = k\pi + \frac{\pi}{6}}$

$2x - \frac{\pi}{2} = 2k\pi + \frac{7\pi}{6} \Rightarrow \boxed{x = k\pi + \frac{5\pi}{6}}$

ب) $\cos(2x - \frac{\pi}{9}) = -\sin 2x \Rightarrow \cos(2x - \frac{\pi}{9}) = \sin(-2x) \rightarrow$

$\cos(2x - \frac{\pi}{9}) = \cos(\frac{\pi}{2} + 2x) \Rightarrow 2x - \frac{\pi}{9} = 2k\pi \pm (\frac{\pi}{2} + 2x) \rightarrow$

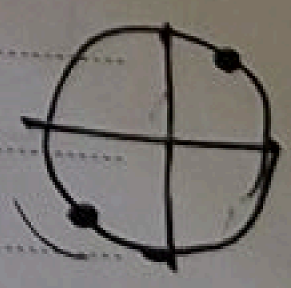
$x = \frac{k\pi}{2} - \frac{7\pi}{72}$

$\frac{\sin 4x + \sin 2x}{\sin 2x} = \sin^2 x + \cos^2 x \Rightarrow \frac{2\sin 2x\cos 2x + \sin 2x}{\sin 2x} = \sin^2 x + \cos^2 x$ ۵

$\frac{\cancel{\sin 2x}(\cos 2x + 1)}{\cancel{\sin 2x}} = 1 \Rightarrow \cos 2x = 0 \Rightarrow 2x = 2k\pi \pm \frac{\pi}{2} \Rightarrow \boxed{x = k\pi \pm \frac{\pi}{4}}$

$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} \Rightarrow \underbrace{\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}}_{\tan\frac{x}{4}} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} \Longrightarrow \tan\frac{x}{2} = \cot\frac{x}{2}$ ۶

$x = \frac{\pi}{2}$ $\quad$ $x = 2k\pi + \frac{\pi}{2}$ $\quad$ $\frac{x}{2} = 2k\pi + \frac{\pi}{4}$



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$\cos^2 x - \sin^2 x \cos 3x = 1$ (۷)

$1 - \cos^2 x = -\sin^2 x \cos 3x$

$\sin^2 x = 0 \Rightarrow \boxed{x = k\pi}$

$-\cos 3x = 1 \Rightarrow 3x = 2k\pi + \pi \Rightarrow \boxed{x = \frac{2k\pi + \pi}{3}}$

$\frac{1 - \tan x}{1 + \tan x} = \tan 3x \Rightarrow \tan(\frac{\pi}{4} - x) = \tan 3x$ (۸)

$3x = k\pi + \frac{\pi}{4} - x \rightarrow 4x = k\pi + \frac{\pi}{4}$

$\Rightarrow x = \frac{k\pi}{4} + \frac{\pi}{16}$

$\sqrt{1 + \sin 2x} = \sqrt{2} \cos 2x$ (۹) $\xrightarrow{\text{توان ۲}} 1 + \sin 2x = 2\cos^2 2x$

$1 + \sin 2x = 2(1 - \sin^2 2x)$

$\Rightarrow 2 - 2\sin^2 2x - 1 - \sin 2x = 0 \Rightarrow -2t^2 - t + 1 = 0$ ($\sin 2x = t$)

$t = -1 \Rightarrow 2x = 2k\pi + \frac{3\pi}{2} \Rightarrow \boxed{x = k\pi + \frac{3\pi}{4}}$

$t = \frac{1}{2} \Rightarrow 2x = 2k\pi + \frac{\pi}{6} \Rightarrow \boxed{x = k\pi + \frac{\pi}{12}}$

$2x = 2k\pi + \frac{5\pi}{6} \Rightarrow \boxed{x = k\pi + \frac{5\pi}{12}}$

$\boxed{x = \frac{3\pi}{4}}$

$\boxed{x = \frac{\pi}{12}}$

$\boxed{x = \frac{5\pi}{12}}$

الف) $\sin^4 x - \cos^4 x = 1 \Rightarrow (\sin^2 x - \cos^2 x)(\cos^2 x + \sin^2 x) = 1$ (۱۰)

$\Rightarrow \cos 2x = -1 = \cos\pi \Rightarrow 2x = 2k\pi \pm \pi$ $\boxed{x = k\pi \pm \frac{\pi}{2}}$

$\Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

ب) $\sin^3 x - \cos^3 x = -1 \Longrightarrow$ نقاط مرزی را در دایره مثلثاتی امتحان می‌کنیم

$\boxed{x = 0, \frac{3\pi}{2}, 2\pi}$