امیرکاشف

۱۷٫۵

اردیبهشت | ۱۸ | 8 May 2022
یکشنبه | ۷ شوال ۱۴۴۳

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الف) $\cos 2x = 3\sin x - 1 \Rightarrow \cos^2 x - \sin^2 x = 3\sin x - 1$

$1 - 2\sin^2 x = 3\sin x - 1 \Rightarrow 2t^2 + 3t - 2 = 0 \rightarrow t = \frac{1}{2}$, $t = -2$ غ ق ق

$\sin x = t$

$\Rightarrow \sin x = 2k\pi + \frac{\pi}{6}$ ، $2k\pi + \pi - \frac{\pi}{6}$

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ب) $\cos 2x + \cos x - 2 = 0 \Rightarrow 2\cos^2 x + \cos x - 3 = 0$

$\cos x = t \Rightarrow 2t^2 + t - 3 = 0$ ، $\frac{-1 \pm \sqrt{1 - 4(-6)}}{4} \Rightarrow t = -\frac{6}{4}$ ✗ ، $t = 1$

$\cos x = \cos 2k\pi \Rightarrow x = 2k\pi$

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الف) $\sin^4 x - \cos^4 x = \sin 4x$

$\Rightarrow \underbrace{(\sin^2 x - \cos^2 x)}_{} \underbrace{(\sin^2 x + \cos^2 x)}_{1} = \sin 4x$

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$\Rightarrow -\cos 2x = 2\sin 2x \cos 2x \Rightarrow \cos 2x (\ldots) = 0$

$\cos 2x = 0 \Rightarrow 2x = k\pi + \frac{\pi}{2} \Rightarrow \boxed{x = \frac{k\pi}{2} + \frac{\pi}{4}}$

$\sin 2x = -\frac{1}{2} \Rightarrow 2x = 2k\pi - \frac{\pi}{6} \Rightarrow \boxed{x = k\pi - \frac{\pi}{12}}$

$2x = 2k\pi + \frac{7\pi}{6} \Rightarrow \boxed{x = k\pi + \frac{7\pi}{12}}$

ب) $2\cos^2 x + \underbrace{2\sin x \cos x}_{\sin 2x} = 1 \Rightarrow \underbrace{2\cos^2 x - 1}_{\cos 2x} = -\sin 2x$

$\Rightarrow \sin\left(\frac{\pi}{2} - 2x\right) = \sin(-2x)$

$\Rightarrow -2x = 2k\pi + \frac{\pi}{2} - 2x \Rightarrow$ ✗

$-2x = 2k\pi + \pi - \frac{\pi}{2} + 2x \Rightarrow -4x = 2k\pi + \frac{\pi}{2} \Rightarrow \boxed{x = -\frac{k\pi}{2} - \frac{\pi}{8}}$

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الف) $\cot x (2\cos x + 1) = \frac{1}{\sin x} \Rightarrow \frac{\cos x}{\sin x}(2\cos x + 1) = \frac{1}{\sin x} \Rightarrow 2\cos^2 x + \cos x = 1$

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$2t^2 + t - 1 = 0$ ، $t = -1 \Rightarrow \cos x = -1 \Rightarrow \boxed{x = 2k\pi + \pi}$

$t = \frac{1}{2} \Rightarrow \cos x = \frac{1}{2} \Rightarrow \boxed{x = 2k\pi \pm \frac{\pi}{3}}$

اگر $\cos x = -1$ باشد $\sin x = 0$ است و عبارت تعریف نشده میشود
پس جواب غیرقابل قبول محسوب میشود

$\tan x = -1 \rightarrow x = k\pi - \frac{\pi}{4}$

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ب) $2\sin^2 x - \sin x\cos x + \cos^2 x = 2 \xrightarrow{\div \cos^2 x} 2\tan^2 x - \tan x + 1 = \frac{2}{\cos^2 x}$ $\quad \frac{1}{\cos^2 x} = 1 + \tan^2 x$

$2\cancel{\tan^2 x} - \tan x + 1 = 2 + \cancel{2\tan^2 x} \Rightarrow \tan x = -1 \rightarrow$ ادامه؟

الف) $\cos(\frac{\pi}{4} + x)\cos(x - \frac{\pi}{4}) = \frac{1}{4}$ ④

$\cos(\frac{\pi}{2} - (\frac{\pi}{4} - x))$

$-\sin(x - \frac{\pi}{4})$

$\Rightarrow \frac{\sin(2x - \frac{\pi}{2})}{-2} = \frac{1}{4} \Rightarrow$

$\sin(2x - \frac{\pi}{2}) = -\frac{1}{2}$ ②

$\Rightarrow 2x - \frac{\pi}{2} = 2k\pi - \frac{\pi}{6}$

$\Rightarrow \boxed{x = k\pi + \frac{\pi}{6}}$

$2x - \frac{\pi}{2} = 2k\pi + \frac{7\pi}{6} \Rightarrow \boxed{x = k\pi + \frac{5\pi}{6}}$

ب) $\cos(2x - \frac{\pi}{9}) = -\sin 2x \Rightarrow \cos(2x - \frac{\pi}{9}) = \sin(-2x) \rightarrow$

$\cos(2x - \frac{\pi}{9}) = \cos(\frac{\pi}{2} + 2x) \Rightarrow 2x - \frac{\pi}{9} = 2k\pi \pm (\frac{\pi}{2} + 2x) \rightarrow$

$x = \frac{k\pi}{2} - \frac{7\pi}{36}$

$\frac{\sin 4x + \sin 2x}{\sin 2x} = \sin^2 x - \cos^2 x \Rightarrow \frac{\sin 2x\cos 2x + \sin 2x}{\sin 2x} = \sin^2 x - \cos^2 x$ ⑤

$\frac{\cancel{\sin 2x}(\cos 2x + 1)}{\cancel{\sin 2x}} = 1 \Rightarrow \cos 2x = 0 \Rightarrow 2x = 2k\pi \pm \frac{\pi}{2} \Rightarrow \boxed{x = k\pi \pm \frac{\pi}{4}}$ ②

$\frac{\sin\frac{\alpha}{2}}{1 + \cos\frac{\alpha}{2}} = \frac{1}{\sin\frac{\alpha}{2}} + \cot\frac{\alpha}{2} \Rightarrow \underbrace{\frac{\sin\frac{\alpha}{2}}{1 + \cos\frac{\alpha}{2}}}_{\tan\frac{\alpha}{4}} = \frac{1 + \cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}} \Rightarrow \tan\frac{\alpha}{4} = \cot\frac{\alpha}{4}$ ⑥

$\alpha = \frac{\pi}{2}$ $\quad \alpha = 2k\pi + \frac{\pi}{2}$ $\quad \frac{\alpha}{4} = k\pi + \frac{\pi}{4}$ ①

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$\cos^2 x - \sin^2 x \cos 3x = 1$

$\frac{1-\cos^2 x}{\sin^2 x} = -\sin^2 x \cos 3x$

$\sin^2 x = 0 \Rightarrow \boxed{x = k\pi}$

$-\cos 3x = 1 \Rightarrow 3x = 2k\pi + \pi \Rightarrow \boxed{x = \frac{2k\pi+\pi}{3}}$

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$\frac{1-\tan x}{1+\tan x} = \tan 3x \Rightarrow \tan(\frac{\pi}{4} - x) = \tan 3x$

$\tan\frac{\pi}{4}$

$3x = k\pi + \frac{\pi}{4} - x \to 4x = k\pi + \frac{\pi}{4}$

$\Rightarrow x = \frac{k\pi}{4} + \frac{\pi}{16}$

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$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \xrightarrow{\text{توان ۲}} 1+\sin 2x = 2\cos^2 2x$

$1+\sin 2x = 2(1-\sin^2 2x) \Rightarrow 2 - 2\sin^2 2x - 1 - \sin 2x = 0 \Rightarrow -2t^2 - t + 1 = 0$ ($\sin 2x = t$)

$t = -1 \Rightarrow 2x = 2k\pi + \frac{3\pi}{2} \Rightarrow \boxed{x = k\pi + \frac{3\pi}{4}}$

$t = -\frac{1}{2} \Rightarrow 2x = 2k\pi - \frac{\pi}{6} \Rightarrow \boxed{x = k\pi - \frac{\pi}{12}}$

$2x = 2k\pi + \frac{7\pi}{6} \Rightarrow \boxed{x = k\pi + \frac{7\pi}{12}}$

$\boxed{x = \frac{3\pi}{4}}$

$\boxed{x = \frac{11\pi}{12}}$

$\boxed{x = \frac{7\pi}{12}}$

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الف) $\sin^4 x - \cos^4 x = 1 \Rightarrow (\sin^2 x - \cos^2 x)(\cos^2 x + \sin^2 x) = $ [illegible]

$\Rightarrow \cos 2x = -1 = \cos\pi \Rightarrow 2x = 2k\pi \pm \pi \quad \boxed{x = k\pi \pm \frac{\pi}{2}}$

ب) $\sin^3 x - \cos^3 x = -1 \Rightarrow$ نقاط مرزی را در بازه می‌گذاریم $\Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\boxed{x = 0, \frac{3\pi}{2}, 2\pi}$

سوال ۶

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \boxed{\tan\frac{x}{4}}$$

$$\frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} = \frac{1}{\tan\frac{x}{4}} = \boxed{\cot\frac{x}{4}}$$

$$\tan\frac{x}{4} = \cot\frac{x}{4} \longrightarrow \tan\frac{x}{4} = \tan\left(\frac{\pi}{2} - \frac{x}{4}\right)$$

$$\frac{x}{4} = k\pi + \frac{\pi}{2} - \frac{x}{4} \rightarrow \frac{x}{2} = k\pi + \frac{\pi}{2} \longrightarrow x = 2k\pi + \pi$$

جواب‌ها

$k=0 \rightarrow x=\pi$

$k=-1 \rightarrow x=-\pi$ $\longrightarrow$ اختلاف $= |\pi - (-\pi)| = 2\pi$

سوال ۷

$$\cos^2 x - \sin^2 x \cos 3x = 1 \xrightarrow{\cos^2 x = 1-\sin^2 x}$$

$$\cancel{1} - \sin^2 x - \sin^2 x \cos 3x = \cancel{1} \longrightarrow \sin^2 x(1+\cos 3x) = 0$$

$\rightarrow \sin x = 0$

$\rightarrow \cos 3x = -1$

$$\sin x = 0 \longrightarrow x = k\pi \longrightarrow x = \overset{3}{0}, \overset{2}{\pi}, \overset{1}{2\pi}$$

$$\cos 3x = -1 \rightarrow 3x = 2k\pi + \pi \rightarrow x = \frac{2k\pi + \pi}{3} \rightarrow x = \overset{5}{\frac{\pi}{3}}, \pi, \overset{4}{\frac{5\pi}{3}}$$

معادله ۵ جواب دارد

سوال ۹

$$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \xrightarrow{\text{توان ۲}} 1+\sin 2x = 2\cos^2 2x$$

$$1+\sin 2x = 2 - 2\sin^2 2x \rightarrow 2\sin^2 2x + \sin 2x - 1 = 0 \rightarrow \sin 2x = -1 \ \text{یا}\ \sin 2x = +\frac{1}{2}$$

$$\sin 2x = -1 \rightarrow 2x = 2k\pi - \frac{\pi}{2} \xrightarrow[k=1]{0 \le x \le \pi} x = \frac{3\pi}{4} \checkmark$$

$$\sin 2x = \frac{1}{2} \rightarrow 2x = 2k\pi + \frac{\pi}{6} \ \text{یا}\ 2k\pi + \frac{5\pi}{6} \xrightarrow[k=0]{0 \le x \le \pi} x = \frac{\pi}{12} \checkmark \ \text{یا}\ \frac{5\pi}{12} \times$$

چون عبارت به توان ۲ رسیده است جواب زائد تولید می‌کند ✓

به ازای $x = \frac{5\pi}{12}$ ، $\cos 2x$ منفی می‌شود، چون جواب رادیکال است غ ق ق است

★ معادله ۲ جواب دارد