۱۵۱۷۵

نام و نام خانوادگی: امیرمحمد بادران‌پور .... پاسخنامه تشریحی تکلیف شماره ...۱۶... کلاس ...دوازدهم تجربی B...

۱

الف) $\cos 2x = 3\sin x - 1 \to 1 - 2\sin^2 x = 3\sin x - 1 \to 2\sin^2 x + 3\sin x - 2 = 0$

$= (\sin x + \frac{4}{2})(\sin x - \frac{1}{2}) = 0$; $\sin x = -2$ غ ق ق ; $\sin x = \frac{1}{2} \to x = 2k\pi + \frac{\pi}{6}$, $x = 2k\pi + \frac{5\pi}{6}$ (۲)

ب) $\cos 2x + \cos x - 2 = 0$; $2\cos^2 x - 1 + \cos x - 2 = 0$; $2\cos^2 + \cos - 3 = 0$

$(\cos + \frac{3}{2})(\cos - \frac{2}{2}) = 0$; $\cos = -1.5$ غ ق ق ; $\cos = 1 \to 2k\pi$

۲

الف) $\sin^4 x - \cos^4 x = \sin 4x \to (\sin^2 x + \cos^2)(\sin^2 x - \cos^2) = 2\sin 2x \cos 2x$

$\sin^2 - \cos^2 = 2 \times \sin x \times \cos x \times (-1)(\sin^2 - \cos^2) = \sin x \times \cos x = -\frac{1}{2} \to$

$\frac{1}{2}\sin 2x = -\frac{1}{2} \to \sin 2x = -1 \to 2k\pi + \frac{3\pi}{2}$ (۱٫۵)

ب) $2\cos^2 x + 2\sin x\cos x - 1 = 0 \to \cos 2x + \sin 2x = 0$; $\cos 2x = -\cos(\frac{\pi}{2} - 2x)$

$2x = -2k\pi - 2x \to 4x = -2k\pi \to x = \frac{-k\pi}{2}$ ✗

$2x = -2k\pi + 2x \to$ ✗

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الف) $\cot x(2\cos x + 1) = \frac{1}{\sin x} \to \frac{\cos x}{\sin x}(2\cos x + 1) = \frac{1}{\sin x}$; $2\cos^2 x + \cos - 1 = 0$

$(\cos x + \frac{2}{2})(\cos x - \frac{1}{2}) = 0$; $\cos x = -1 \to \sin x = 0$ غ ق ق (مخرج صفر) ; $\cos x = \frac{1}{2} \to x = 2k\pi \mp \frac{\pi}{3}$ (۱٫۷۵)

ب) $2\sin^2 x - \sin x\cos x + \cos^2 x = 2 \to 2 - 2\cos^2 x - \sin x\cos x + \cos^2 x - 2 = 0$

$-\cos^2 x - \sin x\cos x = 0$; $-\cos x(\cos x + \sin x) = 0$; $\cos x = 0 \to 2k\pi \mp \frac{\pi}{2}$; $\tan x = -1 \to 2k\pi + \frac{3\pi}{2}$; اشتراک: $2k\pi \pm \frac{\pi}{2}$

$x = k\pi - \frac{\pi}{4}$ و $x = k\pi + \frac{\pi}{2}$

۴

الف) $\cos(\frac{\pi}{4} + x)\cos(x - \frac{\pi}{4}) = \frac{1}{4} \to \cos(\frac{\pi}{4} + x)\sin(\frac{\pi}{2} - (-x + \frac{\pi}{4})) = \frac{1}{4}$

$\frac{1}{2}\sin(\frac{\pi}{2} + 2x) = \frac{1}{4} \to \cos(2x) = \frac{1}{2} \to 2x = 2k\pi \pm \frac{\pi}{3} \to x = k\pi \pm \frac{\pi}{6}$ (۲)

ب) $\cos(2x - \frac{\pi}{9}) = -\sin 2x \to \sin(\frac{\pi}{2} - (2x - \frac{\pi}{9})) = -\sin 2x$

$\sin(-2x + \frac{7\pi}{18}) = -\sin 2x \to \sin(2x - \frac{7\pi}{18}) = \sin 2x$; $2x - \frac{7\pi}{18} = 2k\pi + 2x$ ✗ ; $2x - \frac{7\pi}{18} = 2k\pi + \pi - 2x$ → $x = \frac{k\pi}{2} + \frac{25\pi}{72}$

۵

$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \to \frac{2\sin 2x(\cos 2x + 1)}{\sin 2x} = \cos^2 x + \sin^2 x = 1$ (۱٫۲۵)

$2\cos 2x + 2 = \cos 2x \to \cos 2x = -1$; $2x = 2k\pi + \pi \to x = k\pi + \frac{\pi}{2}$; $2x = 2k\pi - \pi \to x = k\pi - \frac{\pi}{2}$

$x = k\pi \pm \frac{\pi}{2}$

$|\pi - 0| = \boxed{\pi}$ جواب سوال ← π و 0 ← جواب‌ها در بازه $[-\pi, \frac{3\pi}{2}]$

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} \rightarrow \frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} \rightarrow \sin^2\frac{x}{2} = \left(1+\cos\frac{x}{2}\right)^2$$

$$1-\cos^2\frac{x}{2} = \cos^2\frac{x}{2} + 1 + 2\cos\frac{x}{2} \rightarrow 2\cos^2\frac{x}{2} + 2\cos\frac{x}{2} = 0 \rightarrow 2\cos\frac{x}{2}\left(\cos\frac{x}{2}+1\right) = 0$$

$$\cos\frac{x}{2} = 0 \rightarrow \frac{x}{2} = 2k\pi \pm \frac{\pi}{2} \rightarrow x = 4k\pi \pm \pi \rightarrow \boxed{x = 4k\pi + \pi}$$

$$\cos\frac{x}{2} = -1 \rightarrow \frac{x}{2} = 2k\pi + \pi \rightarrow x = 4k\pi + 2\pi \rightarrow \boxed{x = 4k\pi}$$

(۱٫۵)

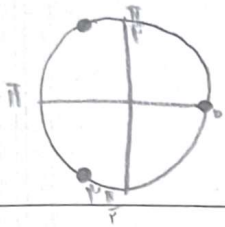
$$\cos^2 x - \sin^2 x \cos 2x = 1 \rightarrow 1 - \sin^2 x - \sin^2 x \cos 2x = 1$$

$$-\sin^2 x\left(1+\cos 2x\right) = 0$$

$\sin^2 x = 0 \rightarrow \sin x = 0 \rightarrow x = 2k\pi$

$\cos 2x = -1 \rightarrow 2x = 2k\pi + \pi \rightarrow x = k\pi + \frac{\pi}{2}$

در بازه $[0, 2\pi]$



← ۴ جواب

(۱٫۵)

$$\frac{1-\tan x}{1+\tan x} = \tan 45^\circ \rightarrow \frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x} = \tan\left(\frac{\pi}{4} - x\right) = \tan 45^\circ$$

$$45^\circ = k\cdot 180^\circ + \frac{\pi}{4} - x$$

$$\boxed{x = k\cdot\frac{\pi}{4} + \frac{\pi}{16}}$$

(۲)

$$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \rightarrow \sqrt{\sin^2 x + \cos^2 x + 2\sin x\cos x} = \sqrt{(\sin x + \cos x)^2}$$

$$|\sin x + \cos x| = \sqrt{2}\left(\cos x - \sin x\right)\left(\cos x + \sin x\right)$$

$\sin + \cos > 0$ ← $\rightarrow \frac{\sqrt{2}}{2} = \cos x - \sin x$

$\sin + \cos < 0$ ← $\rightarrow \frac{\sqrt{2}}{2} = \sin x - \cos x$

(۱)

الف) $\sin^4 x - \cos^4 x = 1 \rightarrow \left(\sin^2 x + \cos^2 x\right)\left(\sin^2 x - \cos^2 x\right) = 1 \rightarrow \cos^2 x - \sin^2 x = 1$

$\cos 2x = -1 \rightarrow 2x = 2k\pi + \pi \rightarrow \boxed{x = k\pi + \frac{\pi}{2}}$

$2x = 2k\pi - \pi \rightarrow \boxed{x = k\pi - \frac{\pi}{2}}$

در بازه $[0, 2\pi]$ → $\frac{\pi}{2}$ و $\frac{3\pi}{2}$

$$\sin^4 x - \cos^4 x = -1 \rightarrow \left(\sin x - \cos x\right)\left(\sin^2 x + \sin x\cos x + \cos^2 x\right) = -1 \quad \left(\sin x - \cos x\right)\left(1 + \frac{\sin 2x}{2}\right) = -1$$

(۱٫۵)

سوال ۲ قسمت ب

$2\cos^2 x + 2\sin x \cos x < 1$ $\begin{cases} 2\sin x\cos x = \sin 2x \\ \frac{1+\cos 2x}{2} = \cos^2 x \end{cases}$

$1 + \cos 2x + \sin 2x < 1 \rightarrow \cos 2x + \sin 2x < 0$

$\tan 2x < -1 \rightarrow 2x = k\pi - \frac{\pi}{4} \rightarrow x = \frac{k\pi}{2} - \frac{\pi}{8}$

سوال ۳

$$\frac{2\sin 2x \cos 2x + \sin 2x}{\sin 2x} = \sin^2 x + \cos^2 x$$

$$\frac{\sin 2x(2\cos 2x + 1)}{\sin 2x} = 1 \xrightarrow{\sin 2x \neq 0} 2\cos 2x + 1 = 1 \rightarrow \cos 2x = 0$$

$2x = k\pi + \frac{\pi}{2} \rightarrow \boxed{x = k\pi + \frac{\pi}{4}}$

سوال ۴

$$\frac{\sin \frac{x}{2}}{1+\cos \frac{x}{2}} = \tan \frac{x}{4}$$

$$\frac{1}{\sin \frac{x}{2}} + \cot \frac{x}{2} = \frac{1+\cos \frac{x}{2}}{\sin \frac{x}{2}} = \frac{1}{\tan \frac{x}{4}} = \cot \frac{x}{4}$$

$\tan \frac{x}{4} = \cot \frac{x}{4} \rightarrow \tan \frac{x}{4} = \tan\left(\frac{\pi}{2} - \frac{x}{4}\right)$

$\frac{x}{4} = k\pi + \frac{\pi}{2} - \frac{x}{4} \rightarrow \frac{x}{2} = k\pi + \frac{\pi}{2} \rightarrow x = 2k\pi + \pi$

جواب‌ها: $k = 0 \rightarrow x = \pi$

$k = -1 \rightarrow x = -\pi$

$\rightarrow$ اختلاف $= |\pi - (-\pi)| = 2\pi$

سوال ۷

$\cos^2 x - \sin^2 x \cos 4x = 1 \xrightarrow{\cos^2 x = 1 - \sin^2 x}$

$1 - \sin^2 x - \sin^2 x \cos 4x = 1 \longrightarrow \sin^2 x (1 + \cos 4x) = 0$

$\sin x = 0$ یا $\cos 4x = -1$

$\sin x = 0 \longrightarrow x = k\pi \longrightarrow x = 0, \pi, 2\pi$ ① ② ③

$\cos 4x = -1 \longrightarrow 4x = 2k\pi + \pi \longrightarrow x = \frac{2k\pi + \pi}{3} \longrightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}$ ④ ⑤

معادله ۵ جواب دارد

سوال ۹

$\sqrt{1 + \sin 4x} = \sqrt{2} \cos 2x \xrightarrow{\text{توان ۲}} 1 + \sin 2x = 2\cos^2 2x$

$1 + \sin 2x = 2 - 2\sin^2 2x \longrightarrow 2\sin^2 2x + \sin 2x - 1 = 0$ $\sin 2x = -1$ یا $\sin 2x = \frac{1}{2}$

$\sin 2x = -1 \longrightarrow 2x = 2k\pi - \frac{\pi}{2} \xrightarrow[k=1]{0 \le x \le \pi} x = \frac{3\pi}{4}$ ✓

$\sin 2x = \frac{1}{2} \longrightarrow 2x = 2k\pi + \frac{\pi}{6}$ یا $2k\pi + \frac{5\pi}{6} \xrightarrow[k=0]{0 \le x \le \pi} x = \frac{\pi}{12}$ یا $\frac{5\pi}{12}$ ✓ ✗

چون عبارت به توان ۲ رسیده است جواب زائد تولید می‌کند

به ازای $x = \frac{5\pi}{12}$ $\cos 2x$ منفی می‌شود پس جواب را نمی‌پذیرد

* معادله ۲ جواب دارد

سوال ۱۰

در سوالات به فرم $\pm\sin^n x \pm \cos^n x$ برابر 1 یا 1- فقط کافی است نقاط اصلی را

ب) چک کنی !

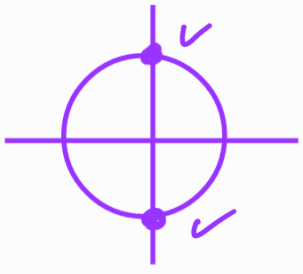
$\sin^3 x - \cos^3 x = 1$

$x = 2\pi$ یا 0

$x = \frac{3\pi}{2}$

الف)

$\sin^4 x - \cos^4 x = 1$



$x = \frac{\pi}{2}$

$x = \frac{3\pi}{2}$