صالح عالی   به نام آنکه جان را فکرت آموخت   دوازده ریاضی B - کلاس 16

۱) الف)

$\cos 2x = 1-2\sin^2 x = 3\sin x - 1 \to 2\sin^2 x + 3\sin x - 2 = 0 \to \sin x = -2$ غ‌ق‌ق ، $\sin x = \frac{1}{2}$ ✓

$$\begin{cases} x = 2k\pi + \frac{\pi}{6} \\ x = 2k\pi + \frac{5\pi}{6} \end{cases}$$

ب) $\cos 2x = 2\cos^2 x - 1 \to 2\cos^2 x - 1 + \cos x - 2 = 0 \to \cos x = -\frac{3}{2}$ غ‌ق‌ق ، $\cos x = 1$ ✓ $\to x = 2k\pi$

۲) الف)

$\sin^4 x - \cos^4 x = (\underbrace{\sin^2 x + \cos^2 x}_{1})(\sin^2 x - \cos^2 x) = -\cos 2x = \sin 4x = 2\sin 2x \cos 2x$

$\to \cos 2x (2\sin 2x + 1) = 0 \to \cos 2x = 0 \to 2x = k\pi + \frac{\pi}{2} \to x = \frac{k\pi}{2} + \frac{\pi}{4}$

$\sin 2x = -\frac{1}{2} \to 2x = 2k\pi + \frac{7\pi}{6} \to x = k\pi + \frac{7\pi}{12}$

$2x = 2k\pi - \frac{\pi}{6} \to x = k\pi - \frac{\pi}{12}$

ب) $\cos 2x + \sin 2x = 0 \to \cos 2x = -\sin 2x \to 2x = k\pi + \frac{3\pi}{4} \to x = \frac{k\pi}{2} + \frac{3\pi}{8}$

۳)

$2\cos x + 1 = \frac{\tan x}{\sin x} \xrightarrow{\sin x \neq 0} 2\cos x + 1 = \frac{1}{\cos x} \to 2\cos^2 x + \cos x - 1 = 0 \to \cos x = -1$ غ‌ق‌ق ← sin را صفر می‌کند ، $\cos x = \frac{1}{2}$

$x = 2k\pi \pm \frac{\pi}{3}$

ب)

$2(1-\cos^2 x) - \sin x \cos x + \cos^2 x = 2 = -\cos^2 x - \sin x \cos x = 0 \to \cos x(\cos - \sin) = 0$

$\to \cos x = 0 \to x = k\pi + \frac{\pi}{2}$

$\to \sin x = \cos x \to x = k\pi + \frac{\pi}{4}$

۴) الف)

$\cos(x - \frac{\pi}{4}) = \cos(\frac{\pi}{4} - x) = \sin(\frac{\pi}{4} + x) \to \cos(\frac{\pi}{4} + x)\sin(\frac{\pi}{4} + x) = \frac{1}{4}$

$\to \frac{1}{2}\sin(\frac{\pi}{2} + 2x) = \frac{1}{4} \to \cos 2x = \frac{1}{2} \to 2x = 2k\pi \pm \frac{\pi}{3} \to x = k\pi \pm \frac{\pi}{6}$

ب)

$\cos(2x - \frac{\pi}{9}) = \sin(\frac{\pi}{2} - 2x + \frac{\pi}{9}) = \sin(-2x + \frac{7\pi}{18}) = -\sin 2x \Rightarrow \sin 2x = \sin(2x - \frac{7\pi}{18})$

$2x - \frac{7\pi}{18} = 2k\pi + 2x \quad \emptyset$ ✗

$\to 2x - \frac{7\pi}{18} = 2k\pi + \pi - 2x \to x = \frac{k\pi}{2} + \frac{25\pi}{72}$

صالح خانلی

۵)
$$\frac{\sin 4x + \sin 2x}{\sin 2x} = \sin^2 x + \cos^2 x = 1 = \frac{\sin 4x}{\sin 2x} + 1 \xrightarrow{\sin 2x \neq 0} \sin 4x = 0 \rightarrow$$

$$4x = k\pi \rightarrow x = \frac{k\pi}{4}$$

$$\sin 2x \neq 0 \rightarrow 2x \neq k\pi \rightarrow x \neq \frac{k\pi}{2}$$

$$\rightarrow x = \frac{k\pi}{2} + \frac{\pi}{4}$$

۸)
$$\frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x} = \tan\left(\frac{\pi}{4} - x\right) = \tan 3x \rightarrow 3x = k\pi + \frac{\pi}{4} - x$$

$$x = \frac{k\pi}{4} + \frac{\pi}{16}$$

۹)
$$\sqrt{\sin^2 x + \cos^2 x + 2\sin x\cos x} = |\sin x + \cos x| = \sqrt{2}(\sin x + \cos x)(\cos x - \sin x)$$

$$\rightarrow \sqrt{2}\cos x - \sqrt{2}\sin x = 1 \rightarrow \frac{\sqrt{2}}{2} = \cos x - \sin x$$

$$\rightarrow \sqrt{2}\cos x - \sqrt{2}\sin x = -1 \rightarrow \frac{\sqrt{2}}{2} = \sin x - \cos x$$

۱۰)

الف) $\underbrace{(\sin^2 x + \cos^2 x)}_{1}(\sin^2 x - \cos^2 x) = 1 \rightarrow \sin^2 x - \cos^2 x = \sin^2 x + \cos^2 x \rightarrow 2\cos^2 x = 0$

$$\cos^2 x = 0 \rightarrow \cos x = 0 \rightarrow x = 2k\pi + \pi$$

ب) $(\sin x - \cos x)(1 + \sin x\cos x) = -1 \rightarrow (\sin x - \cos x)\left(1 + \frac{\sin 2x}{2}\right) = -1$

۷)
$$\cos x - \sin^2 x\cos 3x = \sin^2 x + \cos^2 x \rightarrow \sin^2 x(1 + \cos 3x) = 0$$

$$\rightarrow \sin^2 x = 0 \rightarrow \sin x = 0,\ x = k\pi$$

$$\rightarrow \cos 3x = -1 \rightarrow 3x = 2k\pi + \pi$$

$$x = \frac{2k\pi}{3} + \frac{\pi}{3}$$

جواب ها: $0, \pi, 2\pi, \frac{\pi}{3}, \frac{4\pi}{3}$ ← ۵ تا جواب

۴)
$$\frac{\sin\frac{x}{2}}{1 + \cos\frac{x}{2}} = \frac{1 + \cos\frac{x}{2}}{\sin\frac{x}{2}} \rightarrow \sin^2\frac{x}{2} = \left(1 + \cos\frac{x}{2}\right)^2 \rightarrow 1 - \cos^2\frac{x}{2} = \cos^2\frac{x}{2} + 1 + 2\cos\frac{x}{2}$$

$$2\cos^2\frac{x}{2} + 2\cos\frac{x}{2} = 0 \rightarrow 2\cos\frac{x}{2}\left(\cos\frac{x}{2} + 1\right) = 0$$

$$\rightarrow \cos\frac{x}{2} = 0 \rightarrow \frac{x}{2} = k\pi + \frac{\pi}{2} \rightarrow x = 2k\pi + \pi$$

$$\rightarrow \cos\frac{x}{2} = -1 \quad \text{غ ق ق}$$

در بازه $\left[-\pi, \frac{3\pi}{2}\right]$:

$$|-\pi - \pi| = 2\pi$$

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