۱۷/۵

صالح ...   به نام آنکه جان را فکرت آموخت   دوازده تجربی B - کلاس ۱۶

۱) الف) $\cos 2x = 1 - 2\sin^2 x = 3\sin x - 1 \to 2\sin^2 x + 3\sin x - 2 = 0 \to \sin x = -2$ غ ق ق

$\hookrightarrow \sin x = \frac{1}{2}$ ✓

$$\begin{cases} x = 2k\pi + \frac{\pi}{6} \\ x = 2k\pi + \frac{5\pi}{6} \end{cases}$$

۲

ب) $\cos 2x = 2\cos^2 x - 1 \to 2\cos^2 x - 1 + \cos x - 2 = 0 \to \cos x = -\frac{3}{2}$ غ ق ق

$\hookrightarrow \cos x = 1$ ✓ $\to \cancel{x = 2k\pi}$

۲) الف) $\sin^4 x - \cos^4 x = (\underbrace{\sin^2 x + \cos^2 x}_{1})(\sin^2 x - \cos^2 x) = -\cos 2x = \sin 4x = 2\sin 2x \cos 2x$

$\to \cos 2x(2\sin 2x + 1) = 0 \to \cos 2x = 0 \to 2x = k\pi + \frac{\pi}{2} \to x = \frac{k\pi}{2} + \frac{\pi}{4}$

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$\hookrightarrow \sin 2x = -\frac{1}{2} \to 2x = 2k\pi + \frac{7\pi}{6} \to x = k\pi + \frac{7\pi}{12}$

$\hookrightarrow 2x = 2k\pi - \frac{\pi}{6} \to x = k\pi - \frac{\pi}{12}$

ب) $\cos 2x + \sin 2x = 0 \to \cos 2x = -\sin 2x \to 2x = k\pi + \frac{3\pi}{4} \to x = \frac{k\pi}{2} + \frac{3\pi}{8}$

۳) $2\cos x + 1 = \frac{\tan x}{\sin x} \xrightarrow{\sin x \neq 0} 2\cos x + 1 = \frac{1}{\cos x} \to 2\cos^2 x + \cos x - 1 = 0 \to \cos x = -1$ غ ق ق $\leftarrow$ sin x را صفر می‌کند

$\hookrightarrow \cos x = \frac{1}{2}$

$x = 2k\pi \pm \frac{\pi}{3}$

ج) $2(1 - \cos^2 x) - \sin x \cos x + \cos^2 x = 2 = -\cos^2 x - \sin x \cos x = 0 \to \cos x(\cos - \sin) = 0$

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$\to \cos x = 0 \to x = k\pi + \frac{\pi}{2}$

$\hookrightarrow \sin x = \cos x \to x = k\pi + \frac{\pi}{4}$

۱/۷۵

۴) الف) $\cos(x - \frac{\pi}{4}) = \cos(\frac{\pi}{4} - x) = \sin(\frac{\pi}{4} + x) \to \cos(\frac{\pi}{4} + x)\sin(\frac{\pi}{4} + x) = \frac{1}{4}$

$\to \frac{1}{2}\sin(\frac{\pi}{2} + 2x) = \frac{1}{4} \to \cos 2x = \frac{1}{2} \to 2x = 2k\pi \pm \frac{\pi}{3} \to x = k\pi \pm \frac{\pi}{6}$

ب) $\cos(2x - \frac{\pi}{9}) = \sin(\frac{\pi}{2} - 2x + \frac{\pi}{9}) = \sin(-2x + \frac{7\pi}{18}) = -\sin 2x \to \sin 2x = \sin(2x - \frac{7\pi}{18})$

$2x - \frac{7\pi}{18} = 2k\pi + 2x \quad \emptyset$ ✗

$\to 2x - \frac{7\pi}{18} = 2k\pi + \pi - 2x \to x = \frac{k\pi}{2} + \frac{25\pi}{72}$?

صالح‌دهاش

(۵) $\frac{\sin 4x + \sin 2x}{\sin 2x} = \sin^2 x + \cos^2 x = 1 = \frac{\sin 4x}{\sin 2x} + 1 \xrightarrow{\sin 2x \neq 0} \sin 4x = 0 \to 4x = k\pi \to x = \frac{k\pi}{4}$

$\sin 2x \neq 0 \to 2x \neq k\pi \to x \neq \frac{k\pi}{2}$ $\Big\} \to x = \frac{k\pi}{2} + \frac{\pi}{4}$ (۲)

(۸) $\frac{\tan\frac{\pi}{4} - \tan x}{1 - \tan\frac{\pi}{4}\tan x} = \tan(\frac{\pi}{4} - x) = \tan 3x \to 3x = k\pi + \frac{\pi}{4} - x$

$x = \frac{k\pi}{4} + \frac{\pi}{16}$ (۲)

(۹) $\sqrt{\sin^2 x + \cos^2 x + 2\sin x\cos x} = |\sin x + \cos x| = \sqrt{2}(\sin x + \cos x)(\cos x - \sin x)$

$\to \sqrt{2}\cos x - \sqrt{2}\sin x = 1 \to \frac{\sqrt{2}}{2} = \cos x - \sin x$

یا $\sqrt{2}\cos x - \sqrt{2}\sin x = -1 \to \frac{\sqrt{2}}{2} = \sin x - \cos x$

(۱۰) الف) $\underbrace{(\sin^2 x + \cos^2 x)}_{1}(\sin^2 x - \cos^2 x) = 1 \to \sin^2 x - \cos^2 x = \sin^2 x + \cos^2 x \to 2\cos^2 x = 0$

$\cos^2 x = 0 \to \cos x = 0 \to x = 2k\pi + \pi$ (۱)

ب) $(\sin x - \cos x)(1 + \sin x\cos x) = -1 \to (\sin x - \cos x)(1 + \frac{\sin 2x}{2}) = -1$

(۷) $\cos x - \sin x\cos 3x = \sin^2 x + \cos^2 x \to \sin^2 x(1 + \cos 3x) = 0$ → $\sin^2 x = 0 \to \sin x = 0$, $x = k\pi$ (۲)

یا $\cos 3x = -1 \to 3x = 2k\pi + \pi$

$x = \frac{2k\pi}{3} + \frac{\pi}{3}$

جواب‌ها: $0, \pi, 2\pi, \frac{\pi}{3}, \frac{5\pi}{3}$ ← ۵ تا جواب

(۴) $\frac{\sin\frac{x}{2}}{1 + \cos\frac{x}{2}} = \frac{1 + \cos\frac{x}{2}}{\sin\frac{x}{2}} \to \sin^2\frac{x}{2} = (1 + \cos\frac{x}{2})^2 \to 1 - \cos^2\frac{x}{2} = \cos^2\frac{x}{2} + 1 + 2\cos\frac{x}{2}$

$2\cos^2\frac{x}{2} + 2\cos\frac{x}{2} = 0 \to 2\cos\frac{x}{2}(\cos\frac{x}{2} + 1) = 0$ → $\cos\frac{x}{2} = 0 \to \frac{x}{2} = k\pi + \frac{\pi}{2} \to x = 2k\pi + \pi$ (۴)

یا $\cos\frac{x}{2} = -1$ غ ق ق

در بازه $[-\pi, \frac{3\pi}{2}]$: $|-\pi - \pi| = 2\pi$

سوال ۴ قسمت ب

$$\cos(2x - \frac{\pi}{9}) = -\sin 2x$$

$$\cos(2x - \frac{\pi}{9}) = \cos(\frac{\pi}{2} + 2x)$$

$$2x - \frac{\pi}{9} = 2k\pi \pm (\frac{\pi}{2} + 2x)$$

حالت (+) $\to$ $2k\pi + \frac{\pi}{9} + \frac{\pi}{2} = 0$ $\times$

حالت (−) $\to$ $2x - \frac{\pi}{9} = 2k\pi - \frac{\pi}{2} - 2x$

$$4x = 2k\pi - \frac{7\pi}{18} \to \boxed{x = \frac{k\pi}{2} - \frac{7\pi}{72}}$$

سوال ۹

$$\sqrt{1 + \sin 2x} = \sqrt{2}\cos 2x \xrightarrow{\text{توان ۲}} 1 + \sin 2x = 2\cos^2 2x$$

$$1 + \sin 2x = 2 - 2\sin^2 2x \to 2\sin^2 2x + \sin 2x - 1 = 0$$

$\sin 2x = -1$ و $\sin 2x = +\frac{1}{2}$

$$\sin 2x = -1 \to 2x = 2k\pi - \frac{\pi}{2} \xrightarrow[k=1]{0 \le x \le \pi} x = \frac{3\pi}{4}$$

$$\sin 2x = \frac{1}{2} \to 2x = 2k\pi + \frac{\pi}{6} \text{ یا } 2k\pi + \frac{5\pi}{6} \xrightarrow[k=0]{0 \le x \le \pi} x = \frac{\pi}{12} \text{ یا } \frac{5\pi}{12}$$

چون عبارت به توان ۲ رسیده است جواب زائد تولید می‌کند

به ازای $x = \frac{5\pi}{12}$ $\cos 2x$ منفی می‌شود، پس جواب را باطل است نمی‌تواند است

* معادله ۲ جواب دارد

سوال ۱۰

در سوالات به فرم $\pm\sin^n x \pm \cos^n x$ برابر 1 یا -1 فقط کافی است نقاط اصلی را چک کنی!

ب)

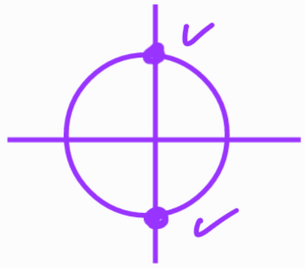
$\sin^3 x - \cos^3 x = 1$

$x = 2\pi$ یا 0

$x = \frac{3\pi}{2}$

الف)

$\sin^4 x - \cos^4 x = 1$



$x = \frac{\pi}{2}$

$x = \frac{3\pi}{2}$