

$$\cos 2\theta = 2\sin \theta - 1$$

$$\sin \theta = -2 \sin \theta + \frac{1}{r}$$

(الف)

$$\Rightarrow 2\sin^2 \theta + 2\sin \theta - 1 = 0 \Rightarrow \sin^2 \theta + 2\sin \theta - \frac{1}{2} = 0$$

$$\sin \theta = \sin \frac{\pi}{5} \Rightarrow \theta = 2K\pi + \frac{\pi}{5} \quad \left(\theta = 2K\pi + (-1)^K \frac{\pi}{5} \right)$$

$$\theta = 2K\pi + \frac{4\pi}{5}$$

$$\cos 2\theta + \cos \theta - 1 = 2\cos^2 \theta + \cos \theta - 1 = 0 \Rightarrow \cos \theta = -\frac{1}{2} \quad \text{و نیز}$$

$$\cos \theta = 1$$

(ب)

$$\cos \theta = \cos 0 \Rightarrow \theta = (2K\pi)$$

$$\sin^2 \theta - \cos^2 \theta = \sin^2 \theta \Rightarrow \sin^2 \theta - \cos^2 \theta = 2\sin^2 \theta \cos^2 \theta \Rightarrow -\cos^2 \theta = 2\sin^2 \theta \cos^2 \theta$$

$$(\sin^2 \theta - \cos^2 \theta)(\sin^2 \theta - \cos^2 \theta) = 2\sin^2 \theta \cos^2 \theta \Rightarrow \sin^2 \theta = -1 \Rightarrow \sin^2 \theta = \frac{1}{r} \Rightarrow \sin \theta = \frac{1}{r}$$

$$\Rightarrow \theta = 2K\pi - \frac{\pi}{5} \Rightarrow \theta = 2K\pi - \frac{\pi}{5} \quad \left(\theta = 2K\pi \pm \frac{\pi}{5} \right)$$

$$2\cos^2 \theta - 1 = -2\sin \theta \cos \theta \Rightarrow \cos 2\theta = -\sin 2\theta$$

(ب)

$$\Rightarrow \cos \theta = -\sin \theta \Rightarrow \theta = 2K\pi - \frac{\pi}{4} \Rightarrow \cos 2\theta = -\sin 2\theta \Rightarrow \theta = 2K\pi - \frac{\pi}{4}$$

$$\frac{\cos \theta}{\sin \theta} = \frac{1}{\sin \theta} \Rightarrow 2\cos^2 \theta + \cos \theta - 1 = 0 \Rightarrow \cos \theta = \frac{1}{2}$$

$$\Rightarrow \theta = (2K\pi + \frac{\pi}{3}) \quad \text{و} \quad \theta = 2K\pi + \frac{\pi}{3}$$

$$2\sin^2 \theta - \sin \theta \cos \theta + \cos^2 \theta = 1 \Rightarrow \sin^2 \theta - \sin \theta \cos \theta = 1$$

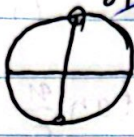
$$\Rightarrow -(\cos^2 \theta - \sin \theta \cos \theta) = 0 \Rightarrow -\cos \theta (\cos \theta + \sin \theta) = 0$$

$$\Rightarrow \cos \theta = 0 \Rightarrow \theta = 2K\pi + \frac{\pi}{2}$$

$$\Rightarrow \cos \theta = -\sin \theta \Rightarrow \theta = 2K\pi - \frac{\pi}{4}$$



$$\frac{\sin^2 x + \sin^2 x}{\sin^2 x} = \sin^2 x \cdot (\cos^2 x) \Rightarrow \frac{\sin^2 x (\cos^2 x + \sin^2 x)}{\sin^2 x} = \sin^2 x + \cos^2 x \quad (a)$$

$$\Rightarrow \frac{\sin^2 x + 1}{\sin^2 x} = 1 \Rightarrow \frac{\sin^2 x}{\sin^2 x} = 0 \quad \Rightarrow \frac{\sin^2 x}{\sin^2 x} = 0 \quad \Rightarrow \frac{\sin^2 x}{\sin^2 x} = 0$$


$$\Rightarrow x = k\pi \pm \frac{\pi}{2}$$

$$\cos^2 x - \sin^2 x \cos^2 x = 1 \Rightarrow -\sin^2 x \cos^2 x = \sin^2 x \quad (v)$$

$$\Rightarrow -\cos^2 x = 1 \Rightarrow \cos^2 x = -1 \Rightarrow x = k\pi + \pi \quad (x = \frac{\pi}{2}, \pi, \frac{3\pi}{2})$$

$$\Rightarrow x = \frac{k\pi}{2} + \frac{\pi}{2}$$

$$(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) = 1 \quad (1. a)$$

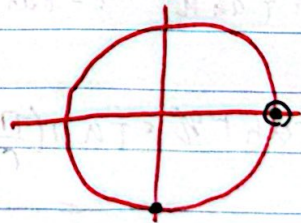
$$\Rightarrow \sin^2 x - \cos^2 x = 1 \Rightarrow -(\cos^2 x) = 1 - \sin^2 x \Rightarrow \cos^2 x = -\cos^2 x$$

$$\Rightarrow \cos^2 x = 0 \Rightarrow \cos x = 0 \Rightarrow x = k\pi \pm \frac{\pi}{2} = (k\pi + \frac{\pi}{2})$$

$$\sin^2 x - \cos^2 x = -1$$

$$\sin^2 x = -1 \Rightarrow x = \left\{ 0, \frac{\pi}{2}, \pi \right\}$$

$$\cos^2 x = 1$$



ب) نقطه نماز

$$\frac{\sin^2 x}{1 + \cos^2 x} = \frac{1 + \cos^2 x}{\sin^2 x} \Rightarrow \sin^2 x \cdot \cos^2 x + 2\cos^2 x + 1 = 0 \quad [-\pi, \frac{\pi}{2}] \quad (b)$$

$$\Rightarrow -\cos^2 x \leq \cos^2 x + 2\cos^2 x \Rightarrow 2\cos^2 x + 2\cos^2 x = 0$$

$$\Rightarrow (2\cos^2 x)(\cos^2 x + 1) = 0 \Rightarrow \cos^2 x = 0 \Rightarrow x = k\pi \pm \frac{\pi}{2} = (k\pi + \frac{\pi}{2})$$

$$\Rightarrow x = k\pi + \pi$$

$$\Rightarrow x = k\pi + \frac{\pi}{2} \Rightarrow x = k\pi + \frac{\pi}{2}$$

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$$\Rightarrow x = \left\{ 0, -\pi, \pi, \frac{\pi}{2} \right\}$$

$$\frac{\sin \frac{x}{r}}{1 + \cos \frac{x}{r}} = \frac{1}{\sin \frac{x}{r}} + \cot \frac{x}{r} \Rightarrow \frac{\sin \frac{x}{r}}{1 + \cos \frac{x}{r}} = \frac{1 + \cos \frac{x}{r}}{\sin \frac{x}{r}} \quad (6)$$

$$\Rightarrow \tan \frac{x}{r} = \frac{1}{\tan \frac{x}{r}} \Rightarrow \tan \frac{x}{r} = \cot \frac{x}{r} \Rightarrow \frac{x}{r} = \frac{\pi}{r} + \frac{\pi}{r}$$

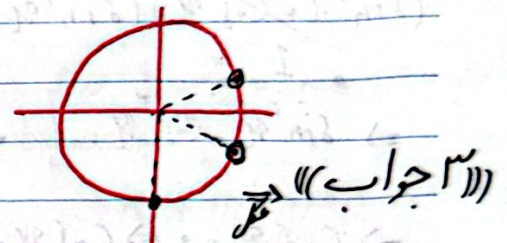
$$\Rightarrow x \in rK\pi + \pi \quad x \in [-\pi, \frac{\pi}{r}] \Rightarrow \{-\pi, \pi\} \Rightarrow |\pi - (-\pi)| = 2\pi$$

$$1 + \sin^2 x = 2 \cos^2 x \quad (9)$$

$$\sin^2 x = 2 \cos^2 x - 1 \Rightarrow \sin^2 x = 1 - 2 \sin^2 x$$

$$\Rightarrow 2 \sin^2 x + \sin^2 x = 1 \Rightarrow \sin^2 x = \frac{1}{3}$$

$$x \in [0, \pi] \quad r x \in [0, 2\pi]$$



$$\frac{1 - \tan x}{1 + \tan x} = \frac{\tan \frac{\pi}{r} - \tan x}{1 - \tan \frac{\pi}{r} \tan x} = \tan(\frac{\pi}{r} - x)$$

$$\Rightarrow \tan^2 x = \tan(\frac{\pi}{r} - x) \Rightarrow rx \in rK\pi + \frac{\pi}{r} - x \Rightarrow x \in \frac{K\pi}{r} + \frac{\pi}{15}$$

$$(\frac{\sqrt{r}}{r} \cos x - \frac{\sqrt{r}}{r} \sin x)(\frac{\sqrt{r}}{r} \cos x + \frac{\sqrt{r}}{r} \sin x) = \frac{1}{r} (\cos^2 x - \sin^2 x) = \frac{1}{r} \quad (10)$$

$$\Rightarrow (\cos^2 x - \sin^2 x) = \frac{1}{r} \Rightarrow 2 \cos^2 x - 1 = \frac{1}{r} \Rightarrow \cos^2 x = \frac{r+1}{2r} \Rightarrow \cos x = \frac{\sqrt{r+1}}{\sqrt{2r}}$$

$$\Rightarrow \cos(r x - \frac{\pi}{9}) = -\sin^2 x = \sin(-2x) = \cos(\frac{\pi}{2} + 2x)$$

$$\Rightarrow \begin{cases} rx - \frac{\pi}{9} = 2K\pi + \frac{\pi}{2} + 2x \\ rx - \frac{\pi}{9} = 2K\pi - \frac{\pi}{2} - 2x \end{cases}$$

$$\Rightarrow rx - \frac{\pi}{9} = 2K\pi - \frac{\pi}{2} - 2x \Rightarrow rx = 2K\pi + \frac{17\pi}{18} \Rightarrow x = \frac{2K\pi}{r} + \frac{17\pi}{18r}$$