به نام خدا
سامان نژادکرد

تکلیف ۱۶

دوازدهم بیسرا B

۲۰ آفرین خیلی خوب نوشتی !!

۱- (۲

$\cos 2x = 3\sin x - 1$

$\Rightarrow 1 - 2\sin^2 x = 3\sin x - 1 \Rightarrow 2\sin^2 x + 3\sin x - 2 = 0 \Rightarrow \begin{cases} \sin x = -2 & \text{غ ق ق} \\ \sin x = \frac{1}{2} \checkmark \end{cases}$

$\Rightarrow \sin x = \sin\frac{\pi}{6} = \frac{1}{2} \Rightarrow x = \begin{cases} 2k\pi + \frac{\pi}{6} \\ 2k\pi + \pi - \frac{\pi}{6} = 2k\pi + \frac{5\pi}{6} \end{cases}$

ب) $\cos^2 x + \cos x - 2 = 0 \Rightarrow 2\cos^2 x + \cos x - 3 = 0$

$a+b+c=0$

$\Rightarrow \begin{cases} \cos x = 1 \Rightarrow x = 2k\pi \\ \cos x = -\frac{3}{2} & \text{غ ق ق} \end{cases}$

۲- (۲

الف) $\sin^4 x - \cos^4 x = \sin 4x$

$\Rightarrow -\cos 2x = 2\sin 2x\cos 2x \Rightarrow 2\sin 2x\cos 2x + \cos 2x = 0$

$\Rightarrow \cos 2x(2\sin 2x + 1) = 0 \Rightarrow \begin{cases} \cos 2x = 0 \Rightarrow 2x = 2k\pi \pm \frac{\pi}{2} \Rightarrow \boxed{x = k\pi \pm \frac{\pi}{4}} \\ \sin 2x = -\frac{1}{2} \Rightarrow 2x = 2k\pi - \frac{\pi}{6} \Rightarrow \boxed{x = k\pi - \frac{\pi}{12}} \end{cases}$

$2x = 2k\pi + \frac{7\pi}{6} \Rightarrow \boxed{x = k\pi + \frac{7\pi}{12}}$

ب) $2\cos^2 x - 1 + 2\sin x\cos x = 0 \Rightarrow \cos 2x + \sin 2x = 0$

$\xrightarrow{\div \cos 2x} 1 + \tan 2x = 0 \Rightarrow \tan 2x = -1 \Rightarrow 2x = k\pi - \frac{\pi}{4} \Rightarrow \boxed{x = \frac{k\pi}{2} - \frac{\pi}{8}}$

۳-

الف) $\frac{\cos x(2\cos x + 1)}{\sin x} = \frac{1}{\sin x} \xrightarrow{\sin x \neq 0} 2\cos^2 x + \cos x = 1$

$\Rightarrow 2\cos^2 x + \cos x - 1 = 0 \Rightarrow \begin{cases} \cos x = -1 & \text{غ ق ق} \Rightarrow \sin x = 0 \\ \cos x = \frac{1}{2} \Rightarrow x = 2k\pi \pm \frac{\pi}{3} \end{cases}$

ب) $\sin^2 x - \sin x\cos x = \underbrace{\sin^2 x + \cos^2 x}_{1} = 1 \Rightarrow -\sin x\cos x = 1 - \sin^2 x$

$\Rightarrow -\cos^2 x = \sin x\cos x \Rightarrow \begin{cases} \cos x = 0 \Rightarrow x = 2k\pi \pm \frac{\pi}{2} \\ -\sin x = \cos x \Rightarrow \tan x = -1 \Rightarrow x = k\pi - \frac{\pi}{4} \end{cases}$

۴- $\text{الف)}\ \cos\left(\frac{\pi}{4}+x\right)\cos\left(x-\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}-x\right)\underbrace{\cos\left(x-\frac{\pi}{4}\right)}_{\cos\left(\frac{\pi}{4}-x\right)}$ ب

$= \frac{1}{2}\sin\left(\frac{\pi}{2}-2x\right) = \frac{1}{2}\cos 2x = \frac{1}{4}$

$\Rightarrow \cos 2x = \frac{1}{2} \Rightarrow 2x = 2k\pi \pm \frac{\pi}{3} \Rightarrow \boxed{x = k\pi \pm \frac{\pi}{6}}$

$\text{ب)}\ \cos\left(2x-\frac{\pi}{9}\right) = -\sin 2x = \sin(-2x) = \cos\left(\frac{\pi}{2}+2x\right)$

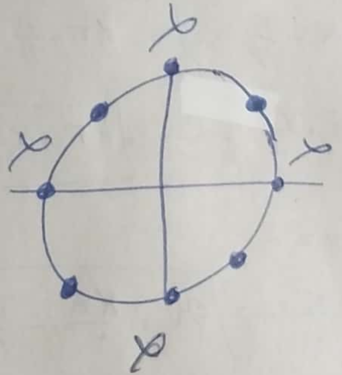
$\Rightarrow \begin{cases} 2x-\frac{\pi}{9} = 2k\pi+\frac{\pi}{2}+2x & \text{غ ق ق} \\ 2x-\frac{\pi}{9} = 2k\pi-\frac{\pi}{2}-2x \Rightarrow 4x = 2k\pi-\frac{7\pi}{18} \Rightarrow x = \frac{k\pi}{2}-\frac{7\pi}{72} \end{cases}$

ب ۵-

$\frac{\sin 4x+\sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x$

$\Rightarrow \frac{\sin 4x+\sin 2x}{\sin 2x} = 1 \Rightarrow \begin{cases}\sin 4x = 0 \\ \sin 2x \neq 0\end{cases} \Rightarrow 4x = \begin{cases}2k\pi+\pi \\ 2k\pi\end{cases} \Rightarrow \begin{cases}x = \frac{k\pi}{2}+\frac{\pi}{4} \\ x = \frac{k\pi}{4}\end{cases}$

$\Rightarrow 2x \neq \begin{cases}2k\pi+\pi \\ 2k\pi\end{cases} \Rightarrow \begin{cases}x \neq k\pi+\frac{\pi}{2} \\ x \neq k\pi\end{cases}$



با توجه به شکل داریم: $x = \frac{k\pi}{2}+\frac{\pi}{4}$

ب ۶-

$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}$

$\Rightarrow \sin^2\frac{x}{2} = 1+\cos^2\frac{x}{2}+2\cos\frac{x}{2} \Rightarrow 1-\cos^2\frac{x}{2} = 1+\cos^2\frac{x}{2}+2\cos\frac{x}{2}$

$\Rightarrow 2\cos^2\frac{x}{2}+2\cos\frac{x}{2} = 0 \Rightarrow \begin{cases}\cos\frac{x}{2} = 0 \\ \cos\frac{x}{2} = -1\end{cases}$ [illegible]: غ ق ق

$\Rightarrow \cos\frac{x}{2} = 0 \Rightarrow \frac{x}{2} = 2k\pi \pm \frac{\pi}{2} \Rightarrow x = 4k\pi \pm \pi$

$\Rightarrow x = -\pi, \pi \Rightarrow |\pi-(-\pi)| = 2\pi$

$$\cos^2 x - \sin^2 x \cos 3x = 1$$ ۷-

$$\Rightarrow \sin^2 x \cos 3x = \cos^2 x - 1 = -\sin^2 x \Rightarrow \sin^2 x \cos 3x + \sin^2 x = 0$$ ۲

$$\Rightarrow \sin^2 x(\cos 3x + 1) = 0 \Rightarrow \begin{cases} \sin^2 x = 0 \Rightarrow \sin x = 0 \Rightarrow \boxed{x = 2k\pi} \\ \cos 3x + 1 = 0 \Rightarrow \cos 3x = -1 \Rightarrow 3x = 2k\pi + \pi \Rightarrow \boxed{x = \frac{2k\pi}{3} + \frac{\pi}{3}} \end{cases}$$

$$\Rightarrow x = \left\{0, 2\pi, \frac{\pi}{3}, \pi, \frac{5\pi}{3}\right\}$$ ۵ جواب

$$\frac{1 - \tan x}{1 + \tan x} = \tan\left(\frac{\pi}{4} - x\right) = \tan 3x$$ ۸-

۲

$$\Rightarrow 3x = k\pi + \frac{\pi}{4} - x \Rightarrow 4x = k\pi + \frac{\pi}{4} \Rightarrow \boxed{x = \frac{k\pi}{4} + \frac{\pi}{16}}$$

توان ۲

$$\sqrt{1 + \sin 2x} = \sqrt{2}\cos 2x \Rightarrow 1 + \sin 2x = 2\cos^2 2x$$ ۹-

$1 + \sin 2x \geq 0$ ✓

$\cos 2x \geq 0$

$$\Rightarrow \sin 2x = \cos\left(\frac{\pi}{2} - 2x\right) = 2\cos^2 2x - 1 = \cos 4x$$

$$\Rightarrow 4x = \begin{cases} 2k\pi + \frac{\pi}{2} - 2x \Rightarrow x = \frac{k\pi}{3} + \frac{\pi}{12} \\ 2k\pi + 2x - \frac{\pi}{2} \Rightarrow x = k\pi - \frac{\pi}{4} \end{cases}$$

۴

$$x = \frac{4k\pi + \pi}{12} \Rightarrow x = \left\{\frac{\pi}{12}, \underbrace{\frac{5\pi}{12}}_{\text{غ ق}}, \frac{3\pi}{4}\right\}$$

$0 \leq x \leq \pi$ $\qquad \cos 2x \geq 0$

$$x = \frac{4k\pi - \pi}{4} \Rightarrow x = \left\{\underbrace{\frac{3\pi}{4}}_{\text{تکراری}},\right\}$$

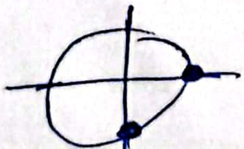
$0 \leq x \leq \pi$

$$x = \left\{\frac{\pi}{12}, \frac{3\pi}{4}\right\}$$ پس تعداد جواب ها ۲

الف) $\sin^4 x - \cos^4 x = (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) = 1$ ۲

$$\Rightarrow -\cos 2x = 1 \Rightarrow \cos 2x = -1 \Rightarrow 2x = 2k\pi + \pi \Rightarrow \boxed{x = k\pi + \frac{\pi}{2}}$$ ۰

ب) $\sin^3 x - \cos^3 x = -1 \Rightarrow \begin{cases} \sin x = -1, \ \cos x = 0 \Rightarrow x = 2k\pi - \frac{\pi}{2} \\ \sin^3 x = 0, \ \cos^3 x = 1 \Rightarrow x = 2k\pi \end{cases}$ چون نقطه اصلا! جا



ادغام

سوال ۱۰

باید جواب ها را در بازه‌ی $[0, 2\pi]$ اعلام کنی!

در سوالات به فرم $\pm \cos^n x \pm \sin^n x$ برابر 1 یا -1 . فقط کافی است نقاط اصلی را چک کنی!

ب)

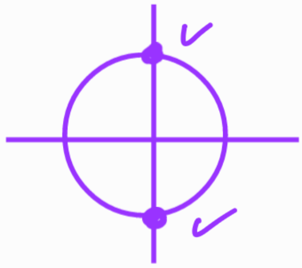
$$\sin^3 x - \cos^3 x = 1$$

$x = 2\pi$ یا 0

$x = \frac{3\pi}{2}$

الف)

$$\sin^4 x - \cos^4 x = 1$$



$x = \frac{\pi}{2}$

$x = \frac{3\pi}{2}$