۱۹ آفرین

« بنام خدا »

« کیوان مکارم شبستر » تکلیف شماره ۱۶

① $k \in \mathbb{Z}$

الف) $\cos 2x = 3\sin x - 1 \xrightarrow{\cos 2x = 1 - 2\sin^2 x} 1 - 2\sin^2 x = 3\sin x - 1$

$\to 2\sin^2 x + 3\sin x - 2 = 0 \to \sin x = \frac{-3 \pm \sqrt{\overset{5}{9 + 16}}}{4}$

$\to \sin x = \frac{-8}{4} = -2$ غیرقابل قبول × $(-1 \le \sin x \le 1)$ ②

$\sin x = \frac{2}{4} = \frac{1}{2} = \sin(\frac{\pi}{6}) \to \boxed{x = 2k\pi + \frac{\pi}{6}}$ ✓ , $x = 2k\pi + \pi - \frac{\pi}{6}$

$\to \boxed{x = 2k\pi + \frac{5\pi}{6}}$ ✓

[redacted] ب) $\cos 2x + \cos x - 2 = 0 \xrightarrow{\cos 2x = 2\cos^2 x - 1} 2\cos^2 x - 1 + \cos x - 2 = 0$

$\to 2\cos^2 x + \cos x - 3 = 0 \xrightarrow[\text{است.}]{\text{جمع ضرایب } 0}$ $\cos x = -\frac{3}{2}$ غیرقابل قبول ✗

$\cos x = 1$

$\to \boxed{x = 2k\pi}$ ✓

② $k \in \mathbb{Z}$

قسمت ب؟ !..

الف) $\sin^4 x - \cos^4 x = \sin 4x \xrightarrow[\text{اتحاد}]{\text{مزدوج}} (\sin^2 x - \cos^2 x)(\cancel{\sin^2 x + \cos^2 x}) = \sin 4x$ ②

$\underset{\text{می دانیم}}{\underline{\cos^2 x - \sin^2 x = \cos 2x}}$ $-\cos 2x = \sin 4x \xrightarrow{\times -} \cos 2x = -\sin 4x \xrightarrow[\text{می دانیم}]{\cos(\frac{\pi}{2} + \alpha) = -\sin\alpha}$

$\cos 2x = \cos(\frac{\pi}{2} + 4x) \to \frac{\pi}{2} + 4x = 2k\pi + 2x \to 2x = 2k\pi - \frac{\pi}{2}$

$\to \boxed{x = k\pi - \frac{\pi}{4}}$ ✓ , $\frac{\pi}{2} + 4x = 2k\pi - 2x \to 6x = 2k\pi - \frac{\pi}{2}$

$\to \boxed{x = \frac{k\pi}{3} - \frac{\pi}{12}}$ ✓

③ الف) $\cot x(2\cos x + 1) = \frac{1}{\sin x} \to \frac{\cos x}{\sin x}(2\cos x + 1) = \frac{1}{\sin x}$ $\quad \cos x = -1$

$\to \frac{2\cos^2 x + \cos x - 1}{\sin x} = 0 \to 2\cos^2 x + \cos x - 1 = 0 \xrightarrow[\text{جمع ضرایب}]{}$ $\cos x = \frac{1}{2}$

① $\cos x = -1 \to \boxed{x = 2k\pi + \pi}$ ✓ , ② $\cos x = \frac{1}{2} \to \boxed{x = 2k\pi \pm \frac{\pi}{3}}$ ✓ ②

ب) $2\sin^2 x - \sin x \cos x + \cos^2 x = 1 + 1 \to -\sin x \cos x = \underbrace{1 - 2\sin^2 x}_{\cos 2x} + \underbrace{1 - \cos^2 x}_{\sin^2 x}$

$-\frac{1}{2}\sin 2x = \cos 2x + \sin^2 x \xrightarrow{\cos 2x = \cos^2 x - \sin^2 x} -\frac{1}{2}\sin 2x = \cos^2 x - \cancel{\sin^2 x} + \cancel{\sin^2 x}$

$\to -\frac{1}{2}\sin 2x = \cos^2 x \to \cos^2 x + \sin x \cos x = 0 \to \cos x(\cos x + \sin x) = 0$

① $\cos x = 0 \to \boxed{x = 2k\pi \pm \frac{\pi}{2}}$ ✓ , ② $\sqrt{2}\sin(x + \frac{\pi}{4}) = 0$

$\to \sin(x + \frac{\pi}{4}) = 0 \to x + \frac{\pi}{4} = k\pi \to \boxed{x = k\pi - \frac{\pi}{4}}$ ✓

$k\in\mathbb{Z}$ ④

الف) $\cos(\frac{\pi}{4}+x)\cos(x-\frac{\pi}{4})=\frac{1}{4}\xrightarrow{\cos(\alpha)=\cos(-\alpha)}\cos(\frac{\pi}{4}+x)\cos(\frac{\pi}{4}-x)=\frac{1}{4}$

$\xrightarrow[\text{متمم}]{\frac{\pi}{4}+x+\frac{\pi}{4}-x=\frac{\pi}{2}}\cos(x-\frac{\pi}{4})=\sin(\frac{\pi}{4}+x)\rightsquigarrow\cos(x+\frac{\pi}{4})\sin(x+\frac{\pi}{4})=\frac{1}{4}$ ②

$\frac{1}{2}\sin(2(x+\frac{\pi}{4}))=\frac{1}{4}\to\sin(\frac{\pi}{2}+2x)=\frac{1}{2}\to\cos 2x=\frac{1}{2}=\cos(\frac{\pi}{3})$

$\to 2x=2k\pi+\frac{\pi}{3}\to\boxed{x=k\pi+\frac{\pi}{6}}$ ✓ , $2x=2k\pi-\frac{\pi}{3}\to\boxed{x=k\pi-\frac{\pi}{6}}$ ✓

ب) $\cos(2x-\frac{\pi}{9})=\sin(-2x)\to\cos(2x-\frac{\pi}{9})=\cos(\frac{\pi}{2}+2x)$

① $2x-\frac{\pi}{9}=2k\pi+\frac{\pi}{2}+2x\to$ جواب ندارد ✗

② $2x-\frac{\pi}{9}=2k\pi-\frac{\pi}{2}-2x\to 4x=2k\pi+\underbrace{\frac{\pi}{9}-\frac{\pi}{2}}_{}$ $-\frac{7\pi}{18}$

$\xrightarrow{\div 4}\boxed{x=\frac{k\pi}{2}-\frac{7\pi}{72}}$ ✓

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⑤ $\frac{\sin 4x+\sin 2x}{\sin 2x}-\sin^2 x=\cos^2 x\xrightarrow{\text{تفکیک}}\frac{\sin 4x}{\sin 2x}+\frac{\sin 2x}{\sin 2x}=\underbrace{\cos^2 x+\sin^2 x}_{1}$ ②

$\frac{2\cancel{\sin 2x}\cos 2x}{\cancel{\sin 2x}}+1=1\to 2\cos 2x=0\to\cos 2x=0\to 2x=2k\pi\pm\frac{\pi}{2}$

$\to\boxed{x=k\pi\pm\frac{\pi}{4}}$ ✓

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⑥ $\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{1}{\sin\frac{x}{2}}+\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\to\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}$, $\underbrace{\frac{\sin\alpha}{1+\cos\alpha}=\frac{1-\cos\alpha}{\sin\alpha}}_{\text{می‌دانیم}}$ ②

$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{1-\cos\frac{x}{2}}{\sin\frac{x}{2}}=\frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}\to\cancel{\sin\frac{x}{2}}-\cos\frac{x}{2}\sin\frac{x}{2}=\cancel{\sin\frac{x}{2}}+\sin\frac{x}{2}\cos\frac{x}{2}$

$\to\sin\frac{x}{2}\cos x+\sin\frac{x}{2}\cos\frac{x}{2}=0\to\sin x=0\to x=k\pi$

$x=\{-\pi,0,\pi\}\to|\pi-0-(-\pi)|=\boxed{2\pi}$ ✓

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⑦ $\cos^2 x-\sin^2 x\cos^3 x=1\to-\sin^2 x\cos^3 x=1-\cos^2 x\to-\sin^2 x\cos^3 x=\sin^2 x$ ②

$\to\sin^2 x\cos^3 x+\sin^2 x=0\to\sin^2 x(\cos^3 x+1)=0$

① $\sin^2 x=0\to\sin x=0\to x=k\pi$ ✓ $\to x=\{0,\pi,2\pi\}$ ✓

② $\cos^3 x+1=0\to\cos 3x=-1\to 3x=2k\pi+\pi\to x=\frac{2k\pi}{3}+\frac{\pi}{3}\to x=\{\frac{\pi}{3},\pi,\frac{5\pi}{3}\}$

درکل ۴ جواب داریم. ✓

★ π بین جوابها مشترک است

ارفاق

(۸)

$\frac{1-\tan x}{1+\tan x} = \tan ۳x \longrightarrow \tan ۳x + \tan ۳x \tan x = 1-\tan x$

$\longrightarrow \tan ۳x + \tan x = 1 - \tan ۳x \tan x \xrightarrow{\tan ۴x = \frac{\tan ۳x + \tan x}{1-\tan ۳x \tan x}} \tan ۴x = 1 = \tan(\frac{\pi}{۴})$ (۲)

$\longrightarrow ۴x = k\pi + \frac{\pi}{۴} \longrightarrow \boxed{x = \frac{k\pi}{۴} + \frac{\pi}{۱۶}}$ ✓ ✓

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(۹)

$\sqrt{1+\sin ۲x} = \sqrt{۲}\cos ۲x \xrightarrow{\text{توان ۲}} 1+\sin ۲x = ۲\cos^{۲} ۲x \longrightarrow ۲\cos^{۲} ۲x - 1 = \sin ۲x$

$\cos ۲x = \sin ۲x \longrightarrow \tan ۲x = 1 \longrightarrow ۲x = k\pi + \frac{\pi}{۴} \longrightarrow \boxed{x = \frac{k\pi}{۲} + \frac{\pi}{۸}}$ (۲)

پس $k = ۰, ۱ \xrightarrow[x\in[۰,\pi]]{} x = \left\{ \frac{\pi}{۸}, \cancel{\frac{\pi}{۲}+\frac{\pi}{۸}} \frac{۵\pi}{۸} \right\}$ ✓ ✓ دو جواب دارد. ✓

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(۱۰)

الف) $\sin^{۴}x - \cos^{۴}x = 1 \xrightarrow{\times -1} \cos^{۴}x - \sin^{۴}x = -1$

مزدوج اتحاد $\underbrace{\cancel{(\cos^{۲}x + \sin^{۲}x)}}_{1}(\cos^{۲}x - \sin^{۲}x) = -1 \xrightarrow{\cos^{۲}x - \sin^{۲}x = \cos ۲x} \cos ۲x = -1$

$\longrightarrow ۲x = ۲k\pi + \pi \longrightarrow x = k\pi + \frac{\pi}{۲} \xrightarrow[k=۰,۱.]{x\in[۰,۲\pi]} x = \left\{ \frac{\pi}{۲}, \cancel{\pi+\frac{\pi}{۲}} \frac{۳\pi}{۲} \right\}$ ✓ (۲)

۲ جواب دارد

ب) $\sin^{۳}x - \cos^{۳}x = -1 \xrightarrow{\tan\frac{x}{۲} = t} \left(\frac{۲t}{1+t^{۲}}\right)^{۳} - \left(\frac{1-t^{۲}}{1+t^{۲}}\right)^{۳} = -1$

پس $t = -1 \longrightarrow \tan(\frac{x}{۲}) = -1 \longrightarrow \frac{x}{۲} = k\pi + \frac{۳\pi}{۴} \longrightarrow x = ۲k\pi + \frac{۳\pi}{۲}$ ✓

$t = ۰ \longrightarrow \tan(\frac{x}{۲}) = ۰ \longrightarrow \frac{x}{۲} = k\pi \longrightarrow x = ۲k\pi$ ✓

$x \in [۰, ۲\pi] \longrightarrow x = \left\{ \frac{۳\pi}{۲}, ۰, ۲\pi \right\}$ ✓

سوال ۲ قسمت ب

$$۲\cos^{۲} x + ۲\sin x \cos x < ۱ \qquad \begin{cases} ۲\sin x \cos x = \sin ۲x \\ \dfrac{۱+\cos ۲x}{۲} = \cos^{۲} x \end{cases}$$

$$۱ + \cos ۲x + \sin ۲x < ۱ \longrightarrow \cos ۲x + \sin ۲x < ۰$$

$$\tan ۲x < -۱ \longrightarrow ۲x = k\pi - \frac{\pi}{۴} \longrightarrow x = \frac{k\pi}{۲} - \frac{\pi}{۸}$$