نام و نام خانوادگی ..[illegible].. پاسخنامه تشریحی تکلیف شماره ..۱۶.. کلاس [illegible] B

۱۹/۷۵ آفرین خیلی خوب نوشتی :))

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الف) $\underbrace{\cos 2x}_{1-2\sin^2 x} = 3\sin x - 1 \to 2\sin^2 x + 3\sin x - 2 = 0$

$\to \sin = -2$ غ ق ق

$\to \sin = \frac{1}{2} \to$

$x = 2k\pi + \frac{\pi}{6}\ ,\ 2k\pi + \frac{5\pi}{6}$

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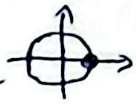
ب) $\underbrace{\cos 2x}_{2\cos^2 x - 1} + \cos x - 2 = 0 \to 2\cos^2 x + \cos x - 3 = 0$

$\to \cancel{\cos x = -\frac{3}{2}}$ غ ق ق

$\to \cos x = +1 \to$



$x = 2k\pi$

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الف) $\underbrace{\sin^4 x - \cos^4 x}_{-\cos 2x} = \underbrace{\sin 4x}_{2\sin 2x \cos 2x} \to -1 = 2\sin 2x \to \sin 2x = -\frac{1}{2} \to 2x = 2k\pi - \frac{\pi}{6}\ ,\ 2k\pi - \frac{5\pi}{6}$

$\to x = k\pi - \frac{\pi}{12}\ ,\ k\pi - \frac{5\pi}{12}$

ب) $\underbrace{2\cos^2 x}_{1+\cos 2x} + \underbrace{2\sin x\cos x}_{\sin 2x} = 1 \to \cos 2x + \sin 2x = 0 \to \cos 2x = -\sin 2x \to \cos 2x = \sin(-2x) \to$

$\to \cos 2x = \cos\left(\frac{\pi}{2} - (-2x)\right) \to 2x = 2k\pi \pm \left(\frac{\pi}{2} + 2x\right) \to 4x = 2k\pi - \frac{\pi}{2}$

$x = \frac{k\pi}{2} - \frac{\pi}{8}$

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الف) $\underbrace{\cot x(2\cos x + 1)}_{\frac{2\cos^2 x + \cos x}{\sin x}} = \frac{1}{\sin x} \to 2\cos^2 x + \cos x - 1 = 0$

$\to \cos x = \frac{1}{2} \to \boxed{x = 2k\pi \pm \frac{\pi}{3}}$

$\to \cos x = -1 \to \boxed{x = 2k\pi + \pi}$ ✗

اگر $\cos x = -1$ باشد، $\sin x = 0$ است و عبارت تعریف نشده می‌شود

پس جواب غیر قابل قبول محسوب می‌شود

ب) $2\sin^2 x - \sin x\cos x + \cos^2 x = 1 \xrightarrow{\div \cos^2 x} \tan^2 x - \tan x = \underbrace{\frac{1}{\cos^2 x}}_{1+\tan^2 x} \to \tan x = -1$

$\to \boxed{x = k\pi - \frac{\pi}{4}}$

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الف) $\cos\left(\frac{\pi}{4} + x\right)\cos\left(x - \frac{\pi}{4}\right) = \frac{1}{4}$; $\cos\left(\frac{\pi}{4} + x\right) = \cos\left(\frac{\pi}{2} + x - \frac{\pi}{4}\right) = -\sin\left(x - \frac{\pi}{4}\right)$

$\to -2\sin\left(2x - \frac{\pi}{2}\right) = 1$; $-2\sin\left(2x - \frac{\pi}{2}\right) = 2\sin\left(\frac{\pi}{2} - 2x\right) = 2\cos(2x)$

$\cos 2x = \frac{1}{2} \to 2x = 2k\pi \pm \frac{\pi}{3} \to \boxed{x = k\pi \pm \frac{\pi}{6}}$

ب) $\cos\left(2x - \frac{\pi}{9}\right) = -\sin 2x = \sin(-2x) = \cos\left(\frac{\pi}{2} + 2x\right)$

$2x - \frac{\pi}{9} = 2k\pi - \frac{\pi}{2} - 2x \to 4x = 2k\pi - \frac{7\pi}{18} \to \boxed{x = \frac{k\pi}{2} - \frac{7\pi}{72}}$

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$\frac{\sin 4x + \sin 2x}{\sin 2x} - \underbrace{\sin^2 x = \cos^2 x} \to \frac{\sin 4x + \sin 2x}{\sin 2x} = 1 \to \sin 4x + \sin 2x = \sin 2x$

$\to \sin 4x = 0 \to 4x = k\pi \to x = \frac{k\pi}{4}$

$\sin 2x \neq 0 \to 2x \neq k\pi \to x \neq \frac{k\pi}{2}$

$\boxed{x = \frac{k\pi}{2} + \frac{\pi}{4}}$

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$\pi-(-\pi)=2\pi$

اختلاف $x_{max}=\pi$, $x_{min}=-\pi$

$$\underbrace{\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}}_{\tan\frac{x}{4}}=\underbrace{\frac{1}{\sin\frac{x}{2}}+\cot\frac{x}{2}}_{\underbrace{\frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}}_{\cot\frac{x}{4}}} \to \tan\frac{x}{4}=\cot\frac{x}{4} \to \frac{x}{4}=\frac{k\pi}{2}+\frac{\pi}{4} \to x=2k\pi+\pi$$

$\sin\frac{x}{2}\neq 0 \to \frac{x}{2}\neq k\pi \to x\neq 2k\pi$

$\cos\frac{x}{2}\neq -1 \to \frac{x}{2}\neq 2k\pi+\pi \to x\neq 4k\pi+2\pi$

$\boxed{x=2k\pi+\pi}$ $\quad x\in[-\pi,\frac{3\pi}{2}]$

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$$\cos^2x-\sin^2x\cos3x=1 \to \sin^2x(1+\cos3x)=0$$

$\cancel{\cos^2x+\sin^2x}$

$\cos3x=-1 \to 3x=2k\pi+\pi$

$\sin x=0 \to \boxed{x=k\pi \quad , \quad x=\frac{2k\pi}{3}+\frac{\pi}{3}}$

$x\in[0,2\pi]$

$\boxed{\underbrace{\{0,\pi,2\pi,\frac{\pi}{3},\frac{5\pi}{3}\}}_{\text{۵جواب}}}$

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$$\frac{1-\tan x}{1+\tan x}=\tan2x \to \frac{\frac{\cos x-\sin x}{\cancel{\sin x}}}{\frac{\cos x+\sin x}{\cancel{\sin x}}}=\frac{\sin2x}{\cos2x} \to$$

$$\to \sin2x\cos x+\underbrace{\sin2x\sin x-\cos2x\cos x}_{-\cos4x}+\cos2x\sin x=0 \to \underbrace{\sin4x}_{\cos(\frac{\pi}{2}-4x)}=\cos4x$$

$\sin4x$

$$4x=2k\pi+\frac{\pi}{2}-4x \to 8x=2k\pi+\frac{\pi}{2} \to \boxed{\frac{k\pi}{4}+\frac{\pi}{16}=x}$$

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$$\underbrace{\sqrt{1+\sin2x}}_{\underbrace{\sqrt{\sin^2x+2\sin x\cos x+\cos^2x}}_{\underbrace{\sin x+\cos x}_{\sqrt{2}\sin(x+\frac{\pi}{4})}}}=\sqrt{2}\cos2x \to \cancel{\sqrt{2}}\underbrace{\sin(x+\frac{\pi}{4})}_{\cos(\frac{\pi}{4}-x)}=\cancel{\sqrt{2}}\cos2x$$

$2x=2k\pi+\frac{\pi}{4}-x \to 3x=2k\pi+\frac{\pi}{4} \to \boxed{x=\frac{2k\pi}{3}+\frac{\pi}{12}}$

$2x=2k\pi-\frac{\pi}{4}+x \to \boxed{x=2k\pi-\frac{\pi}{4}}$

$x\in[0,\pi]$

$x=\underbrace{\{\frac{\pi}{12},\frac{9\pi}{12}\}}_{\text{۲جواب}}$

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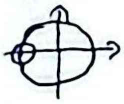
الف) $\underbrace{\sin^4x-\cos^4x}_{-\cos2x}=1 \to \cos2x=-1 \to 2x=2k\pi+\pi \to \boxed{x=k\pi+\frac{\pi}{2}}$



$x\in[0,2\pi]$

$\boxed{x=\underbrace{\{\frac{\pi}{2},\frac{3\pi}{2}\}}_{\text{۲جواب}}}$

ب) $\sin^3x-\cos^3x=-1$ با انتهای زوایای مهم

$0,2\pi$

$\frac{3\pi}{2}$

$\boxed{x=\underbrace{\{0,\frac{3\pi}{2},2\pi\}}_{\text{۳جواب}}}$

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