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Year ..... Month ..... Day ..... Subject .....

«به نام خدا»

تکلیف شماره ۱۵ — کیوان مکارم نسب

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① $x=0 \rightarrow y=\tan(\frac{\pi}{4})=1 \rightarrow B=\begin{bmatrix}0\\ \frac{\pi}{4}\end{bmatrix}$

مختصات B $\begin{bmatrix}0\\1\end{bmatrix}$ است پس ارتفاع هم ۱ است

می دانیم $\tan\alpha \rightarrow \alpha \neq \frac{\pi}{2} \rightarrow -2x+\frac{\pi}{4} \neq \frac{\pi}{2} \rightarrow 2x \neq -\frac{\pi}{4} \rightarrow x \neq -\frac{\pi}{8}$

$\rightarrow A=\begin{bmatrix}-\frac{\pi}{8}\\ 0\end{bmatrix}$, $-2x+\frac{\pi}{4} \neq -\frac{\pi}{2} \rightarrow 2x \neq \frac{3\pi}{4} \rightarrow x \neq \frac{3\pi}{8}$

$\rightarrow C=\begin{bmatrix}\frac{3\pi}{8}\\ 0\end{bmatrix} \rightarrow \overline{AC}=\frac{3\pi}{8}-(-\frac{\pi}{8})=\frac{4\pi}{8}=\frac{\pi}{2}$, $\overline{OB}=$ ارتفاع $=\cancel{\frac{\pi}{4}}$ 1

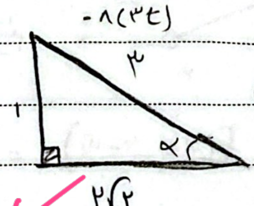
$\rightarrow S_{ABC}^{\triangle}=\frac{1}{2}AC\times OB=\frac{1}{2}\times\frac{\pi}{2}\times\cancel{\frac{\pi}{4}}=\boxed{\cancel{\frac{\pi^2}{16}}}$ ✓ $\frac{\pi}{4}$

② می دانیم : $1-\sin^2\alpha=\cos^2\alpha \rightarrow x^2-(\sin\alpha)x-\frac{1}{4}(1-\sin^2\alpha)=0$

$\rightarrow x=\frac{2}{3} \rightarrow \frac{4}{9}-\frac{2}{3}\sin\alpha-\frac{1}{4}(1-\sin^2\alpha)=0 \rightarrow \frac{4}{9}-\frac{2}{3}\sin\alpha-\frac{1}{4}+\frac{\sin^2\alpha}{4}=0$

$\xrightarrow{\times 36} 16-24\sin\alpha-9+9\sin^2\alpha=0 \xrightarrow{\sin\alpha=t} 9t^2-\underbrace{24t}_{-8(3t)}+7=0$

$\rightarrow (3t-1)(3t-7)=0 \xrightarrow{-1\leq\sin\alpha\leq1} \sin\alpha=\frac{7}{3}$ غیرقابل قبول

$\sin\alpha=\frac{1}{3}$ قابل قبول $\rightarrow$  $\cos\alpha=\frac{2\sqrt{2}}{3}$

$\rightarrow 1+\tan^2\alpha=\frac{1}{\cos^2\alpha}=\frac{1}{\frac{8}{9}}=\frac{9}{8}$ ✓

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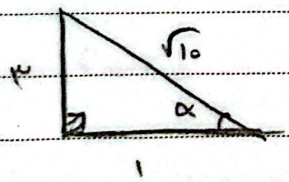
③ $\log(\sin\alpha)-\log(-\cos\alpha)=\log 3 \xrightarrow[\text{لگاریتم}]{\text{ویژگی}} \log\frac{-\sin\alpha}{\cos\alpha}=\log 3$

پایه لگاریتم برابر از ۱ است $\rightarrow -\frac{\sin\alpha}{\cos\alpha}=3 \rightarrow \tan\alpha=-3$ (باتوجه به اینکه دامنه لگاریتم از

۰ تا مثبت بی نهایت است: $\sin\alpha>0$ و $-\cos>0$، درنتیجه $\cos\alpha<0$ است).

طبق توضیحات زاویه α در ناحیه دوم است.



$\rightarrow \sin\alpha=\frac{3}{\sqrt{10}}$, $\cos\alpha=-\frac{1}{\sqrt{10}}$

$\rightarrow \sin\alpha-\cos\alpha=\frac{3}{\sqrt{10}}-(-\frac{1}{\sqrt{10}})=\frac{4}{\sqrt{10}}=\frac{4\sqrt{10}}{10}=\frac{2\sqrt{10}}{5}$

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$\sin^2\theta = 1-\cos^2\theta \longrightarrow (\sin^2\theta)^2+\cos^4\alpha = \frac{4}{5}$

$\longrightarrow (1-\cos^2\theta)^2+\cos^4\theta = \frac{4}{5} \longrightarrow \cos^4\theta - 2\cos^2\theta + 1 + \cos^4\theta = \frac{4}{5}$

$\xrightarrow{\cos\theta = t} t^4 - t^2 + 1 = \frac{4}{5} \xrightarrow{\times 5} 5t^4 - 5t^2 + 5 = 4$

$\longrightarrow 5t^4 - 5t^2 + 1 = 0 \longrightarrow t^2 = \frac{5 \pm \sqrt{25-20}}{10} \longrightarrow t^2 = \frac{5\pm\sqrt5}{10}$

$\longrightarrow t_1^2 = \frac{5-\sqrt5}{10} \xrightarrow{①} \cos^4\theta + \sin^4\theta = (\cos^2\theta)^2 + 1 - \cos^2\theta = ?$

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$\longrightarrow t_2^2 = \frac{5+\sqrt5}{10} \xrightarrow{②} \cos^4\theta + \sin^4\theta = ((\cos^2\theta)^2 + 1 - \cos^2\theta = ?$

① $\left(\frac{5-\sqrt5}{10}\right)^2 + 1 - \frac{5+\sqrt5}{10} = \frac{30-10\sqrt5}{100} + 1 - \frac{5+\sqrt5}{10} = \frac{3-\sqrt5}{10} + \frac{-5-\sqrt5}{10} + 1$

$= \frac{8-2\sqrt5}{10} + 1 = \frac{4-\sqrt5}{5} + \frac{5}{5} = \frac{9-\sqrt5}{5}$ ✓

② $\left(\frac{5+\sqrt5}{10}\right)^2 + 1 - \frac{5+\sqrt5}{10} = \frac{30+10\sqrt5}{100} + 1 + \frac{-5-\sqrt5}{10} = \frac{3+\cancel{\sqrt5}}{10} + \frac{-\cancel{\sqrt5}-5}{10} + 1$

$\frac{-2}{10} + 1 = 1 - \frac{1}{5} = \boxed{\frac{4}{5}}$ ✓

همواره مثبت

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$\frac{\sin x\,(3+\cos x)}{2(1-\cos^2x)} < 0 \longrightarrow \frac{\sin x\,(3+\cos x)}{2\sin^2 x} < 0 \longrightarrow \frac{\boxed{3+\cos x}}{2\sin x} < 0$

$\longrightarrow \sin x < 0$ ① , $\sin x \cdot \frac{\sin x}{\cos x} - \frac{1}{\cos x} > 0 \longrightarrow \frac{\sin^2 x - 1}{\cos x} > 0$

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$\frac{-\cos^2 x}{\cos x} > 0 \longrightarrow -\cos x > 0 \longrightarrow \cos x < 0$ ②

①و② ← ناحیه سوم ✓

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$\frac{2}{\sin x} = \frac{-3}{\cos x} \longrightarrow \frac{\sin x}{\cos x} = -\frac{2}{3} = \tan x$

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$\longrightarrow \cot x = -\frac{3}{2} \longrightarrow \tan x + \cot x = -\frac{2}{3} - \frac{3}{2} = \frac{-4-9}{6} = \boxed{\frac{-13}{6}}$ ✓

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$\frac{\sqrt{1+\cos 40}}{\cos 40 + \cos 80} = \frac{\sqrt{2\cos^2 20}}{\cos 40 + \cos 80} = \frac{\sqrt2\,\cos 20}{\cos 40 + \cos 80} = \frac{\sqrt2\,\cos 20}{\cancel{\cos 40 + 2\cos^2 40 - 1}}$

$(2\cos 40 - 1)(\underbrace{\cancel{\cos 40 + 1}}_{2\cos^2 20})$

$\longrightarrow \frac{\sqrt2}{2(\cos 20)(2\cos 40 - 1)} = \frac{\sqrt2}{2(\cos 20)(2(2\cos^2 20 - 1) - 1)} = \frac{\sqrt2}{2}\,\frac{1}{2(4\cos^3 20 - 3\cos 20)}$

$\cos 3x = \cos 60$ ✓

$\longrightarrow \frac{\sqrt2}{2} \times \frac{1}{2\cos 60} = \frac{\sqrt2}{\frac{2}{\sqrt3}} = \frac{\sqrt2}{2\sqrt3} = \frac{\sqrt6}{6}$ ✓

داریم : $1-2\sin^2\alpha=\cos2\alpha$, $2\cos^2\alpha-1=\cos2\alpha$, $\frac{1-\tan^2\alpha}{1+\tan^2\alpha}=\cos2\alpha$ ۸

$\Rightarrow (\cos10^\circ)(\cos20^\circ)(\cos40^\circ)=?$, $2\sin\alpha\cos\alpha=\sin2\alpha$

$\xrightarrow{\times\frac{2\sin10^\circ}{2\sin10^\circ}} \frac{(2\sin10^\circ\cos10^\circ)(\cos20^\circ)(\cos40^\circ)}{2\sin10^\circ}=\frac{\times2\sin20^\circ\cos20^\circ\cos40^\circ}{2\times2\sin10^\circ}=\frac{\sin40^\circ\cos40^\circ}{4\sin10^\circ}$ ۲

$\xrightarrow[\times2]{\times2}\frac{2\sin40^\circ\cos40^\circ}{8\sin10^\circ}=\frac{\sin80^\circ}{8\sin10^\circ}\xrightarrow{\sin80^\circ=\cos10^\circ}\frac{1}{8}\frac{\cos10^\circ}{\sin10^\circ}=\boxed{\frac{1}{8}\cot10^\circ}$ ✓

$\sin\alpha+3\cos\alpha=2\xrightarrow{\text{توان }2}\sin^2\alpha+9\cos^2\alpha+6\sin\alpha\cos\alpha=4$ ۹

$\xrightarrow{\div\sin^2\alpha} 1+9\cot^2\alpha+6\cot\alpha=\frac{4}{\cancel{\sin^2\alpha}}$ $(1+\cot^2\alpha)$ $\xrightarrow{\cot\alpha=t} 1+9t^2+6t=4(1+t^2)$

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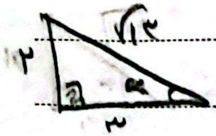
$9t^2+6t+1=4+4t^2\to 5t^2+6t-3=0\to t=\frac{-6\pm\sqrt{36+45}}{10}$ (9)

$\to \cot\alpha=\frac{3}{10}$, $\cot\alpha=-\frac{15}{10}=-\frac{3}{2}$



$\to \cos2\alpha=2\left(\frac{9}{13}\right)-\frac{13}{13}=\frac{5}{13}\Rightarrow 5\left(\frac{5}{13}\right)-12\left(\frac{3}{\sqrt{13}}\right)=\frac{25}{13}-\frac{36}{\sqrt{13}}$ ✓

$\to \frac{25-36\sqrt{13}}{13}$ ✓

$\cancel{\hat B}+\hat C+\hat A=180^\circ$ (72°) $\longrightarrow \hat A+\hat C=108^\circ \longrightarrow \frac{\cancel{\sin72}}{\sin108}\sin(108-\hat A)=\frac{1+\cos A}{2}$ ۱۰

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$(\sin108)(\sin108\cos A-\cos108\sin A)=\frac{1+\cos A}{2}$

$\sin^2108^\circ\cos A-\frac{1}{2}\sin216\sin A=\frac{1+\cos A}{2}$

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$$\sin\alpha + 3\cos\alpha = 2 \rightarrow \sin\alpha = 2 - 3\cos\alpha \xrightarrow{\text{توان ۲}} \sin^2\alpha = (2-3\cos\alpha)^2$$

$$1-\cos^2\alpha = 9\cos^2\alpha + 4 - 12\cos\alpha \rightarrow 10\cos^2\alpha - 12\cos\alpha + 3 = 0 \xrightarrow{\text{فرمول طلایی}}$$

$$10\left(\frac{1+\cos 2\alpha}{2}\right) - 12\cos\alpha + 3 \rightarrow 5 + \underbrace{5\cos 2\alpha - 12\cos\alpha}_{\text{(خواسته سوال)}} + 3 = 0$$

$$5\cos 2\alpha - 12\cos\alpha = \boxed{-8}$$

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$$\cos^2\frac{A}{2} = \frac{1+\cos A}{2} \rightarrow 2\sin B\sin C = 1+\cos\hat{A} \xrightarrow{A+B+C=\pi} A = \pi - (\hat{B}+\hat{C})$$

$$1+\cos(\pi-(B+C)) = 1-\cos(B+C) = 2\sin B\sin C$$

$$1-\cos B\cos C + \sin B\sin C = 2\sin B\sin C \rightarrow \sin B\sin C + \cos B\cos C = 1$$

$$\cos(B-C) = 1 \rightarrow \begin{cases} B-C=0 \ \checkmark \\ B-C=2\pi \ \times \ \text{مثلث} \end{cases} \rightarrow B=C \rightarrow A = 180^\circ - 144^\circ = 36^\circ$$

$$B+C = 2B \quad (2\times 72)$$

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$$\sqrt{1+\sin 50^\circ} = \sqrt{1+\cos 40^\circ} \xrightarrow[1+\cos 40^\circ = 2\cos^2 20^\circ]{\text{فرمول طلایی}} \sqrt{2\cos^2 20^\circ} = \sqrt{2}\,|\cos 20^\circ| = \sqrt{2}\cos 20^\circ$$

$$\sin 50^\circ + \sin 10^\circ = \sin(30^\circ+20^\circ) + \sin(30^\circ-20^\circ) =$$

$$\sin 30^\circ\cos 20^\circ + \sin 20^\circ\cancel{\cos 30^\circ} + \sin 30^\circ\cos 20^\circ - \cancel{\sin 20^\circ}\cos 30^\circ = 2\left(\frac{1}{2}\right)\cos 20^\circ = \cos 20^\circ$$

$$\frac{\sqrt{2}\cos 20^\circ}{\cos 20^\circ} = \boxed{\sqrt{2}}$$