۱۹/۷۵ آفرین

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①

$\tan(3x + \frac{\pi}{4}) = \tan(\frac{\pi}{4}) = 1 = BH$

$T = \frac{\pi}{|3|} = \frac{\pi}{3} = AC$ ✓ $\quad S_{ABC} = \frac{AC \times BH}{2} = \frac{\frac{\pi}{3} \times 1}{2} = \frac{\pi}{6}$ ✓

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②

$x^2 - \sin a\, x - \frac{1}{4}\cos^2 a = 0 \xrightarrow{x = 2/3}$

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$\frac{4}{9} - \frac{2}{3}\sin a - \frac{1}{4}(1 - \sin^2) = 0$

در فرمول های مثلثاتی دقت کن!

$\frac{4}{9} - \frac{2}{3}\sin a - \frac{1}{4} + \frac{1}{4}\sin a^2 = 0 \xrightarrow{\text{طرفین } \times 36} 16 - 24\sin a + 9\sin^2 - 9$

$\longrightarrow 9\sin a^2 - 24\sin a + 7 = 0 \xrightarrow{ac} \sin a^2 - 24\sin a + 63 = 0 \longrightarrow (\sin - 3)(\sin - 21)$

$\longrightarrow \sin a = 1/3 \qquad 1 + \tan^2 = \frac{1}{\sin a^2}$ (cos²a) $\Rightarrow 1 + \tan^2 = 9$

$\cos^2 a = 1 - \frac{1}{9} = \frac{8}{9}$

$1 + \tan^2 a = \frac{9}{8}$

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③

$\log \sin a - \log -\cos a = \log 3 \longrightarrow \log \frac{\sin a}{-\cos a} = \log 3$

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$\frac{\sin a}{-\cos a} = 3 \longrightarrow \sin a = -3\cos a \longrightarrow \sin^2 + \cos^2 = 1 = 9\cos a^2 + \cos^2 = 10\cos^2 = 1$

$(-3\cos a)$

$\longrightarrow \cos = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10} \longrightarrow \sin a = \frac{-3\sqrt{10}}{10}$

$\frac{-3\sqrt{10}}{10} - \frac{\sqrt{10}}{10} = \frac{-4\sqrt{10}}{2} \longrightarrow \sin a - \cos a = \frac{-2\sqrt{10}}{10}$ ✓

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④

$\sin^4 + \cos^2 = 4/5 \qquad \cos^4 + \sin^2 = \cos^4 - \cos^2 + 1 = -\sin^2\cos^2 + 1$

$(1 - \sin^2) \qquad (1 - \cos^2) \qquad \cos^2(\cos^2 - 1) \to -\sin^2$

$\sin^4 - \sin^2 + 1 = 4/5 = -1/5$

$\sin^2(\sin^2 - 1) = -1/5 \longrightarrow \sin^2\cos^2 = 1/5 \qquad \cos^4 + \sin^2 = -1/5 + 1 = 4/5$ ✓

$(\sin^2 - 1 \to -\cos^2)$

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⑤

$\frac{3\sin x + \sin x \cos x}{2 - 2\cos^2} < 0 \qquad \frac{\sin(3 + \cos x)}{2(1 - \cos^2)} = \frac{\sin(3 + \cos x)}{2\sin^2}$ (sin)

همواره +

$\frac{3 + \cos}{2\sin} < 0 \Longrightarrow \sin x =$ منفی است

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− −	+ −

همواره ⊕ ـ ناحیه سوم (− −) ✓

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$\sin x \tan x - \frac{1}{\cos} = \frac{\sin^2}{\cos} \cdot \frac{1}{\cos} = \frac{\sin^2 - 1}{\cos} > 0 \longrightarrow \cos x$ منفی است

همواره منفی است چون تا به لذا داخل

(6)

$\frac{2}{\sin x}+\frac{3}{\cos x}=0$

$\frac{3\sin x+2\cos x}{\sin x\cos x}=0 \longrightarrow 3\sin x+2\cos x=0 \longrightarrow 3\sin x=-2\cos x$

$\longrightarrow -\frac{3}{2}\sin x=\cos x$

$\sin^2+\cos^2=1 - \sin^2 x+\frac{9}{4}\sin^2 x=1 \to \frac{13}{4}\sin^2 x=1$

$\sin x^2=4/13 \longrightarrow \frac{2\sqrt{13}}{13}\sin x \;,\; \cos x=\frac{-3\sqrt{13}}{13}$ ✓

$\text{Tan}\,x+\cos x=\frac{1}{\sin x\cos x} \Rightarrow \frac{1}{\frac{2\sqrt{13}}{13}\times\frac{-3\sqrt{13}}{13}}=\frac{13}{-6}$ ✓

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(7)

$\frac{\sqrt{1+\sin 50^\circ}}{\sin 50^\circ+\sin 10^\circ}$

$\sin 50^\circ=\cos 40^\circ \longrightarrow \sqrt{1+\cos 40^\circ}=\sqrt{2}\cos 20^\circ$ ✓

$2\cos 20^{\circ 2}$ ← فرمول طلایی ✓

2

$\sin(30+20)+\sin(30-20)=\sin 30^\circ\cos 20^\circ+\sin 20^\circ\cos 30^\circ+\sin 30^\circ\cos 20^\circ-\sin 20^\circ\cos 30^\circ$

$=2\sin 30^\circ\cos 20^\circ=\cos 20^\circ$

$\longrightarrow \frac{\sqrt{2}\cos 20^\circ}{\cos 20^\circ}=\sqrt{2}$ ✓

(8)

$(1-2\sin 5^{\circ 2})(2\cos 10^{\circ 2}-1)\left(\frac{1-\tan 20^{\circ 2}}{1+\tan 20^{\circ 2}}\right)=?=\cos 10^\circ\cos 20^\circ\cos 40^\circ$

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$\cos 10$, $1/2\sin 20^\circ\cos 20^\circ$, $\frac{\cos 10}{1/2\sin 80^\circ}$

$\Rightarrow \frac{\sin 10^\circ}{\sin 10^\circ}\times\cos 10^\circ\times\cos 20^\circ\times\cos 40^\circ=1/8\,\frac{\sin 80^\circ}{\sin 10^\circ}=1/8\cot 10^\circ$ ✓

$1/2\sin 40^\circ$

(9)

$\sin+3\cos=2 \longrightarrow \sin=2-3\cos \qquad \sin^2=4-12\cos+9\cos^2$

$1-\cos^2$

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$10\cos^2-12\cos+3=0 \longrightarrow 10\cos^2-12\cos x=-3$

$5\cos 2x-12\cos=10\cos^2-12\cos-5=? \to -8$ ✓

$2\cos^2-1$, (-3)

10

$\sin B\sin C=\cos^2\frac{A}{2} \xrightarrow{\text{فرمول طلایی}} 2\sin B\sin C=\cos A+1$

$A=180^\circ-(B+C) \longrightarrow \cos A=\cos(180^\circ-(B+C))$

$2\sin B\sin C=1-\cos(B+C)$

$2\sin B\sin C=1-\cos B\cos C+\sin B\sin C$

$\sin B\sin C=1-\cos B\cos C$

$\cos B\cos C+\sin B\sin C=1 \qquad \cos(B-C)=1$

$B=C=72^\circ \Rightarrow A=36^\circ$ ✓

2