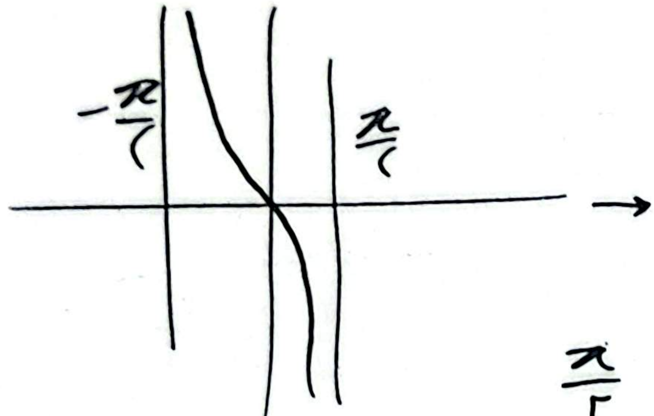
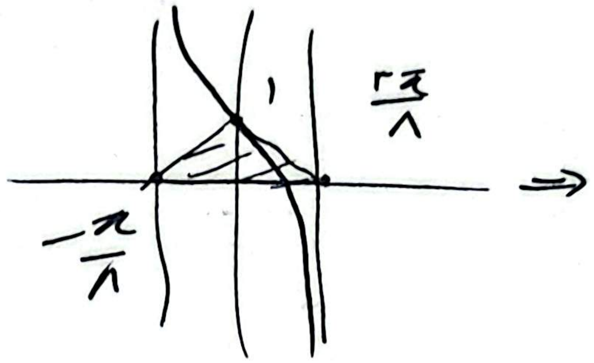
شعبه گلبیت: ۱۵۲ — سجاد سجادی

$y = -\tan(2x - \frac{\pi}{4})$ ①





$S = \frac{1}{2} \times 1 \times \frac{4\pi}{8} = \frac{\pi}{4}$ (with $\frac{4\pi}{8}$ struck out and $\frac{\pi}{2}$ written above it)

②

$\frac{4}{9} - \frac{2}{3}\sin\alpha - \frac{1}{4}\cos^2\alpha = 0 \Rightarrow$ (with $\cos^2\alpha$ struck out and $1-\sin^2\alpha$ written above it)

$\frac{1}{4}\sin^2\alpha - \frac{2}{3}\sin\alpha + \frac{4}{9} - \frac{1}{4} = 0 \Rightarrow \begin{cases} \sin\alpha = \frac{7}{3} \ ✗ \\ \sin\alpha = \frac{1}{3} \ ✓ \end{cases}$ (with $\frac{4}{9} - \frac{1}{4}$ struck out and $\frac{7}{36}$ written below it)

$\sin\alpha = \frac{1}{3} \rightarrow \cos^2\alpha = \frac{8}{9} \Rightarrow$

$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} = \frac{1}{\frac{8}{9}} = \boxed{\frac{9}{8}}$

(۳)

$tg = \frac{\sin\alpha}{\cos\alpha}$, $tg = -3 \Rightarrow -\cos\alpha = 3 \Rightarrow \boxed{\cos\alpha = -3}$

شرط‌های خود سوال: $\sin\alpha > 0$, $\cos\alpha < 0$ $\Big\} \to$ ناحیه دوم $\to$ $1+\tan^2\alpha = \frac{1}{\cos^2\alpha} \Rightarrow$

$\cos\alpha = \frac{-1}{\sqrt{10}}$, $\sin\alpha = \frac{3}{\sqrt{10}}$

$\to \sin\alpha - \cos\alpha = \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{10}} = \frac{4}{\sqrt{10}}$

(۴)

$\sin^4\theta - \sin^2\theta + \frac{1}{5} = 0$

$\cos^4\theta + \sin^2\theta = (1-\sin^2\theta)^2 + \sin^2\theta = \sin^4\theta - 2\sin^2\theta + 1 + \sin^2\theta$

$= \sin^4\theta - \sin^2\theta + 1 = \boxed{\sin^4\theta - \sin^2\theta + \frac{1}{5}} + \frac{4}{5} = \boxed{\frac{4}{5}}$

(۵)

$\frac{\sin x(3+\cos x)}{2\sin^2 x} < 0 \to \boxed{\sin x < 0}$

$\sin x\cos x - \frac{\sin^2 x + \cos^2 x}{\cos x} = \sin x\cos x - \sin x\cos x - \cos x > 0 \to \boxed{\cos x < 0}$

$\to$ پس در ناحیه سوم (۳) قرار دارد

$$\frac{\Sigma}{\sin^2\theta} + \frac{9}{\cos^2\theta} + \frac{12}{\sin\theta\cos\theta} \cdots \Rightarrow \quad (4)$$

$$\Sigma(1+\cot^2\theta) + 9(1+\tan^2\theta) + 12(\tan\theta+\cot\theta) \cdots \Rightarrow$$

$$\Sigma\cot^2\theta + 12\cot\theta + 9 + 9\tan^2\theta + 12\tan\theta + \Sigma \cdots \Rightarrow$$

$$(3\cot\theta + 3)^2 + (3\tan\theta + 3)^2 \cdots \Rightarrow$$

$$\cot\theta = -\frac{3}{3} \qquad \tan\theta = -\frac{3}{3} \rightarrow \cot\theta + \tan\theta = -\frac{12}{9}$$

$$\frac{\sin 20 + \cos 20}{\sin(30+30) + \sin(60-30)} = \frac{\sqrt{2}\sin(45)}{\cos 30} = \sqrt{2} \quad (5)$$

$$(1-2\sin^2 0)\left(2\cos^2 10 - 1\right)\left(\frac{1-\tan^2 20}{1+\tan^2 20}\right) \times \frac{\sin 10}{\sin 10} = \quad (6)$$

$$\frac{1}{4}\cdot\frac{\sin A}{\sin 10}\cdot\cos 10 = \frac{1}{4}\cot 10$$

$\sin\alpha < 2 - 3\cos\alpha \xrightarrow[2]{\text{بتربیع}} \sin^2\alpha < 9\cos^2\alpha - 12\cos\alpha + 4 \longrightarrow$ (9)

$1 - \cos^2\alpha$

$10\cos^2\alpha - 12\cos\alpha + 3 > 0$

حواستون باشه : $a(2\cos^2\alpha - 1) - 12\cos\alpha < 10\cos^2\alpha - 12\cos\alpha - 5$

$10\underbrace{\cos^2\alpha - 12\cos\alpha + 3}_{0} - 8 < 1 - 8$

$\cos A < 108 \rightarrow \hat{C} < 108 - \hat{A}$ (10)

$\sin 72^\circ \underbrace{\sin(108^\circ - \hat{A})}_{\sin(180^\circ - (108 - \hat{A}))} = \sin 72 \sin(72 + A) = \sin 72 (\sin 72\cos A + \sin A \cos 72)$

$\sin^2 72\cos A + \sin 72\cos 72 \sin A < \frac{1 + \cos A}{2} \Rightarrow$

$2\sin^2 72\cos A + \sin 144 \sin A < 1 + \cos A \longrightarrow$

$\cos A(\underbrace{2\sin^2 72 - 1}_{-\cos 144}) < 1 - \sin 144 \sin A \Rightarrow$

$-1 < \cos A \cos 144 - \sin A \sin 144 \Rightarrow$

$-1 < \cos(A + 144) \rightarrow \boxed{A < 36^\circ}$