تکلیف ۱۴ – نویدامینی

۱۷

۱)

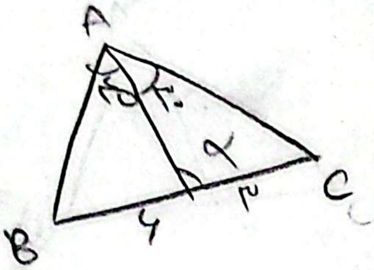
$BT = \tan(\frac{\pi}{2}-\alpha) = \cot\alpha \to OB = \sqrt{1+\cot^2\alpha} = \frac{1}{\sin\alpha} \to AB = OB - OA$

$\frac{1}{\sin\alpha} - 1 = \frac{1-\sin\alpha}{\sin\alpha}$

۲

---

۲)



$\frac{3}{\sin 30} = \frac{AC}{\sin\alpha} \to AC = 6\sin\alpha$

$\frac{6}{\sin 45} = \frac{AB}{\sin(\pi-\alpha)} \to AB = 6\sqrt{2}\sin\alpha$

$\to \frac{AB}{AC} = \frac{6\sqrt{2}\sin\alpha}{6\sin\alpha} = \sqrt{2}$

۲

---

۳)

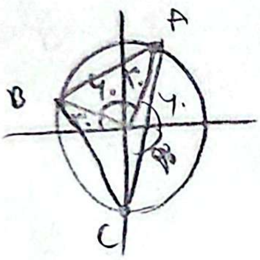
$A: (\frac{1}{2}, \frac{\sqrt{3}}{2})$

$B: (-\frac{\sqrt{3}}{2}, \frac{1}{2})$

$C: (0, -1)$

$S = \frac{1}{2}\left| \dots \right| = \left(\frac{1}{4}+\frac{\sqrt{3}}{2} - \left(-\frac{3}{4}-\frac{1}{2}\right)\right) \times \frac{1}{2} = \frac{3+\sqrt{3}}{4} \to$ الف

۱,۲,۳



$AB = 2r\sin\frac{90}{2} = 2\times1\times\frac{\sqrt{2}}{2} = \sqrt{2}$

$AC =$ $BC^2 = 1^2+1^2 - 1\times1\times2\times-\frac{1}{2} = 1+1+1 = 3 \to BC = \sqrt{3}$

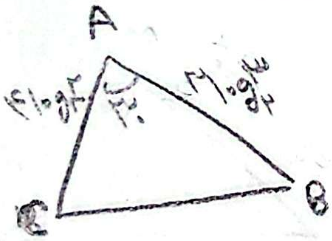
$AC^2 = 1^2+1^2 - 1\times1\times2\times-\frac{\sqrt{3}}{2} = 1+1+\sqrt{3} \to AC = \sqrt{2+\sqrt{3}}$

$AB^2 = 1^2+1^2 \to AB = \sqrt{2}$

$P_{ABC} = \sqrt{2}+\sqrt{3}+\sqrt{2+\sqrt{3}}$

---

۴)



$S = \frac{1}{2}\times\frac{1}{2}\times 4\log_3^2 \times 3\log_8^3 = \frac{3\log_2^3}{\log_2^3} = 3$

۲

---

۵)

$x^2(2x^2-1) - \sin^2 x(2\sin^2 x-1) = f(x)$

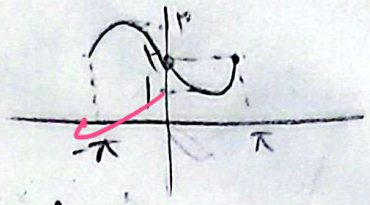
$x^2(2x^2-1) - \sin^2\alpha(2\sin^2\alpha-1) = x - \cos(g(x))+1$

$f(\cos\alpha) = \cos^2\alpha(\underbrace{2\cos^2\alpha-1}_{\cos2\alpha}) - \sin^2\alpha(\underbrace{2\sin^2\alpha-1}_{-\cos2\alpha}) = \cos2\alpha(\underbrace{\cos^2\alpha+\sin^2\alpha}_{1}) = 1 \to \cos2\alpha = 1$

$\to \sin2\alpha = 0$

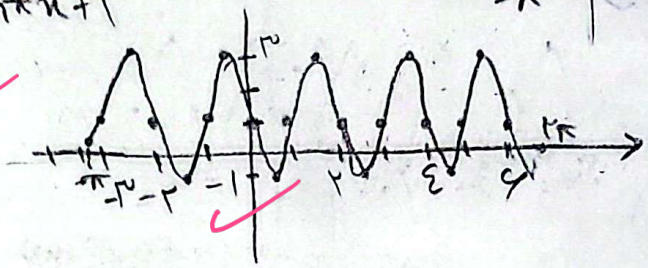
۲

(۴) $-۲\sin x\cos x+۲=-\sin ۲x+۲$ $T=\frac{۲\pi}{۲}=\pi$ ✓



(۲) $۲\cos(۳x+\frac{\pi}{۲})+۱=-۲\sin ۳x+۱$

$T=\frac{۲\pi}{۳}$ ✓

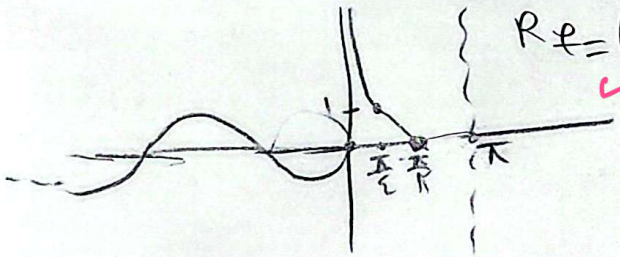


$[-\pi,\pi]\cong[-۳,۶]$

---

(۷) $f(x)=\begin{cases}\cot x & ;0<x<\frac{\pi}{۲}\\ \sin x & ;x\le 0\end{cases}$ $\cot x=\tan(\frac{\pi}{۲}-x)$

(۲)



$R_f=(-۱,+\infty)$

---

(۸) (۱,۷۵)

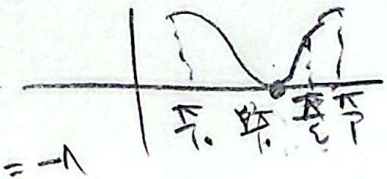
$f(x)=a+b\sin(x)\cos(cx)(\cos(۲cx)=a+\frac{b}{۲}\sin(۲cx)\cos(۲cx)=a+\frac{b}{۴}\sin(۴cx)$

$T=\frac{\frac{\pi}{۲}-\frac{\pi}{۱۰}}{۱}=\frac{۵\pi-\pi}{۱۰}=\frac{۴\pi}{۱۰}=\frac{۲\pi}{۴|c|}\to |c|=۵\xrightarrow{c>0}c=۵$

$a+|\frac{b}{۴}|=۶$

$a-|\frac{b}{۴}|=۰\to a=۳,\ |\frac{b}{۴}|=۳$

$f'(x)=-۴c\frac{b}{۴}\sin ۵cx=bc\sin ۵cx=bc\sin ۵\alpha x$



$f'(\frac{\pi}{۴})>۰\to b\sin\frac{۵\pi}{۴}>۰\to b<۰\to\frac{b}{۴}=-۲\to b=-۸$

$-\frac{۴\pi}{۲}\times\frac{a}{۲}=-۵$

$a+\frac{b}{۴}=۴$

$a-\frac{b}{۴}=۰\to a=۲,\ b=۸$

---

(۹) $|a|+c=۲$

$-|a|+c=-۲\to c=-۱$

$|a|=۳$

$\frac{۲\pi}{|b|}=\pi\to|b|=۲\to f(x)=-۳\cos ۲x-۱$

$f'(۰)>۰\to a\neq -۳$

$-۳\cos ۲\alpha-۱=۰$

$\to\cos ۲\alpha=-\frac{۱}{۳}$

$\tan\alpha$

$۱+\tan^۲ ۲\alpha=۹\to\tan\alpha=\pm\frac{۱}{۲}$

$$f(-۲۴,۵) = f(-۲۴+۱۵) = f(۲(-۱۲)+۱۵) = f(۱۵)$$

$$f(x) \begin{cases} -\frac{۱}{۲}x+۱ & ; \cdot \leq x \leq ۲ \\ ۲x+۲ & ; -۱ \leq x < \cdot \end{cases} \longrightarrow f(۱,۵) = -\frac{۱}{۲}\left(\frac{۳}{۲}\right)+۱ = \frac{۱}{۴}$$