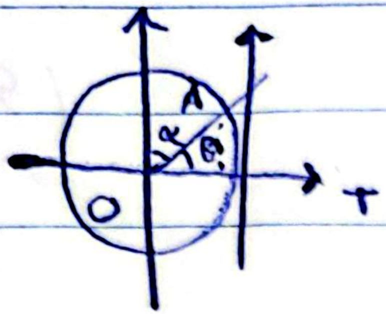
۱)



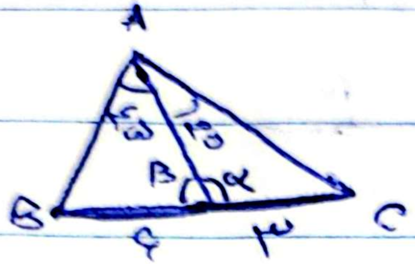
$T_B = \tan\beta = \cot\alpha$

$OA = r = 1 , OT = 1$

$OB^۲ = (OT)^۲ + (TB)^۲ \longrightarrow OB = \sqrt{\underbrace{1+\cot^۲\alpha}_{\frac{1}{\sin^۲\alpha}}}$

$OB - OA = \frac{1}{\sin\alpha} - 1$

۲)



$\sin\alpha = \sin\beta$

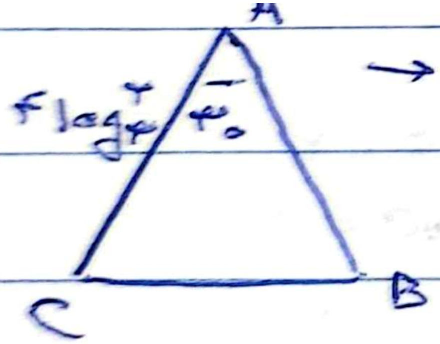
$\alpha + \beta = ۱۸۰$

$\frac{۳}{\sin ۳۰} = \frac{AC}{\sin\alpha}$ و $\frac{۶}{\sin ۴۵} = \frac{AB}{\sin\beta} \Rightarrow \frac{AB}{AC} = \frac{۶\sqrt{۲}}{۶} \Rightarrow \frac{AB}{AC} = \sqrt{۲}$

۳)

$AC = \sqrt{۲+\sqrt{۳}} = \frac{\sqrt{۲}+\sqrt{۶}}{۲}$ $\quad BC = \sqrt{۳}$ $\quad P = \frac{۳\sqrt{۲} + ۲\sqrt{۳} + \sqrt{۶}}{۲}$

$\sin A \times \frac{1}{۲} \times AC \times AB \Rightarrow \frac{\sqrt{۳}}{۲} \times \frac{1}{۲} \times \frac{\sqrt{۲}+\sqrt{۶}}{۲} \times \sqrt{۲} = \frac{۳+\sqrt{۳}}{۴}$



$\rightarrow \log_{۳۰}^{۲} = t$

$CB = \sqrt{۱۶t^{۲} + c^{۲} - \sqrt{۴۸}\,tc}$

(۴

$\Rightarrow \dfrac{a}{t^{۲}} = ۱۶t^{۲} - ۴\sqrt{۳}\,tc \Rightarrow a^{۲} = b^{۲} + c^{۲} - ۲bc\cos A$

$\Rightarrow c = \underline{b\sqrt{۳} \pm \sqrt{۴a^{۲} - b^{۲}}}$

$S = \sin A \times \dfrac{۱}{۲} \times b \times c = \dfrac{bc}{۴}$

$\dfrac{۴\sqrt{۳}\,t \pm \sqrt{۴۹ - ۱۶t^{۲}}}{۲}$

$S = ۲\sqrt{۳}\,(\log_{۳۰}^{۲})^{۲} \pm \sqrt{۹ - ۴(\log_{۳۰}^{۲})^{۲}}$

(۵

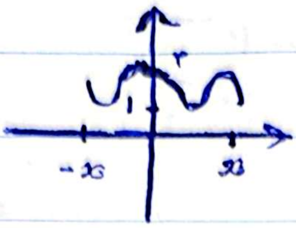
$۲x^{۴} = x^{۲} + \sin^{۲}(\alpha)(\cos ۲\alpha) \Longrightarrow \cos^{۲}\alpha(\cos ۲\alpha) + \sin^{۲}\alpha(\cos ۲\alpha)$

$\cos ۲\alpha = 1 \Longrightarrow ۲\alpha = x \Longrightarrow \frac{x}{۲} = \alpha \Longrightarrow \sin ۲\alpha = 0$

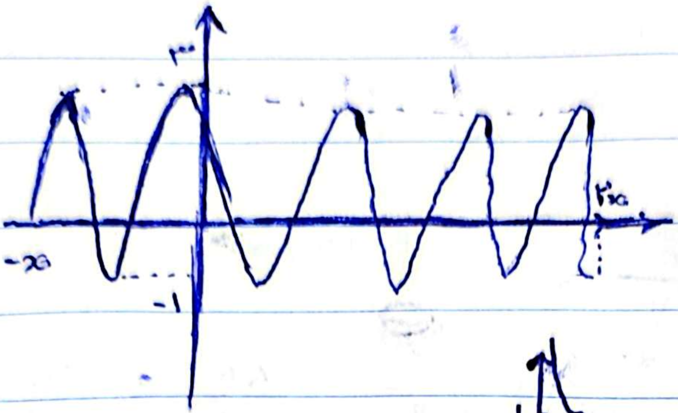
A

۶)

الف) $T=\pi$

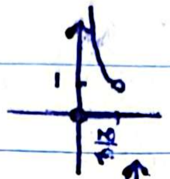


ب) $T=۲$



۷)

$g(x)\begin{cases} \cot x & : \quad ۰<x<\frac{\pi}{۴} \Rightarrow \\ \sin x & : \quad x\leq ۰ \Rightarrow \end{cases}$

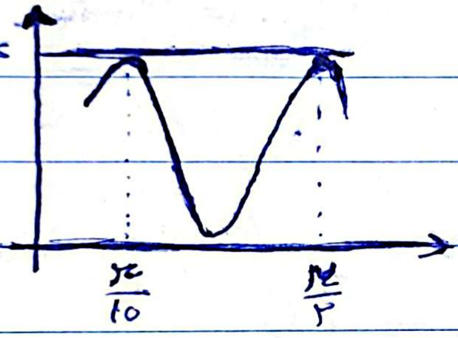


$R\in(۱,+\infty)$

$R\in[-۱,۱]$

$R\in[-۱,+\infty)$

۸)



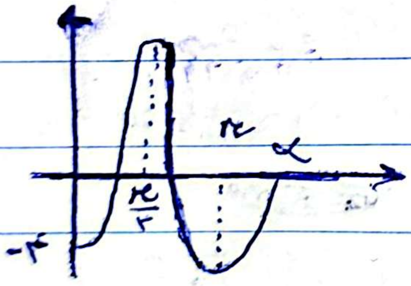
$a+\frac{b}{۴}\sin(۴c\pi)$

$T=\frac{۲\pi}{|۴c|}=\frac{۲\pi}{۵}\Rightarrow ۴c=۵\rightarrow c=\frac{۵}{۴}$

$\left.\begin{matrix} a+|\frac{b}{۴}|=۴ \\ a+|\frac{b}{۴}|=۰ \end{matrix}\right\} a=۲ \text{ و } \frac{b}{۴}=۲\Rightarrow b=۸$

$\frac{bc}{a}=۵$

۹)



$T=\pi\Rightarrow b=۲$

$|a|+c=۲$

$-|a|+c=-۴$

$c=-۱$

$|a|=۳\Rightarrow a=\pm۳$

$-۳\cos ۲\alpha-۱=۰\Rightarrow -\frac{۱}{۳}=\cos ۲\alpha$

$|\tan ۲\alpha|=۲\sqrt{۲}$

۱۵)

$g(-۳۴٫۵) = \frac{۱}{۴}$

$y = -\frac{۱}{۲}x + ۱ \Rightarrow x = \frac{۳}{۲}$

$y = \frac{۱}{۴}$