به نام خدا

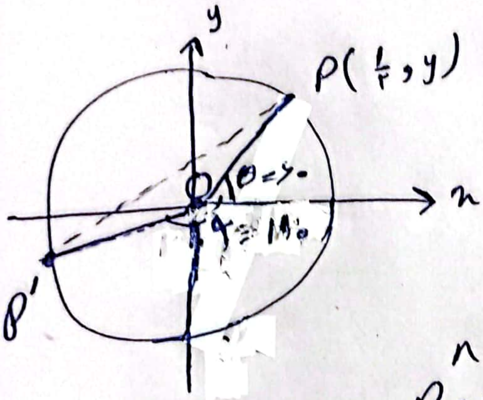
سما ن نژادکرد　　　　دوازدهم تجربی B　　　　تکلیف ۱۲

۱- $$\frac{a\sin\frac{4\pi}{3} + b\cos\frac{7\pi}{6}}{a\sin\frac{11\pi}{6} + b\cos\frac{\pi}{3}} = \frac{-\frac{\sqrt{3}}{2}a - \frac{\sqrt{3}}{2}b}{-\frac{1}{2}a + \frac{1}{2}b} = \frac{-\frac{\sqrt{3}}{2}(a+b)}{-\frac{1}{2}(a-b)} = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \frac{-\sqrt{3}(a+b)}{a-b} = \frac{\sqrt{3}}{2} \Rightarrow \frac{a+b}{a-b} = \frac{-1}{2} \Rightarrow 2a+2b = b-a$$

$$\Rightarrow 2b = -6a$$

$$\Rightarrow \frac{b}{a} = \frac{-6}{2} = -3$$

۲- 

$$P(\tfrac{1}{2}, y) \Rightarrow x_p = r\cos\theta = 1\cos\theta$$

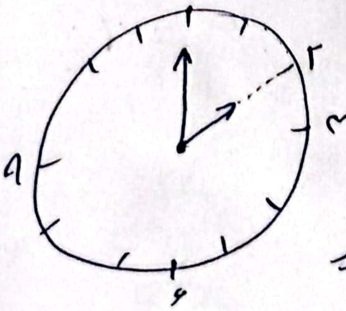
$$\Rightarrow \cos\theta = \frac{1}{2} \left\{ \Rightarrow \theta = 60^\circ \xRightarrow{210^\circ \text{ دوران}} \theta' = -150^\circ \right.$$

θ در ربع اول

$$P\hat{O}P' = 360 - 60 - 150 = 150^\circ$$

$$\Rightarrow PP' = \sqrt{OP^2 + OP'^2 - 2\,OP\,OP'\cos 150^\circ} = \sqrt{2+\sqrt{3}}$$

$$\Rightarrow \text{محیط} = P = OP + OP' + PP' = 2 + \sqrt{2+\sqrt{3}}$$

۳- 

از آنجایی که کل ساعت ۷۲۰ دقیقه و 2π رادیان است

پس داریم: $\frac{2\pi}{12\times 60'} = \frac{\frac{\pi}{9}}{x} \Rightarrow x = 40'$

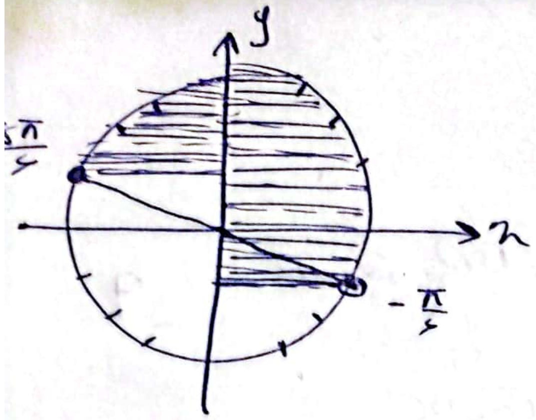
پس با $\frac{\pi}{9}$ حرکت عقربه ساعت‌شمار $40'$ طی می‌شود

پس ساعت $2:40'$ است.

۴- $$|5.5M - 30H| = 20 \Rightarrow |5.5M - 90| = 20$$

$$\Rightarrow \begin{cases} 5.5M - 90 = 20 \Rightarrow 5.5M = 110 \Rightarrow \boxed{M = 20} \Rightarrow 3:20' \\ 5.5M - 90 = -20 \Rightarrow 5.5M = 70 \Rightarrow M = \frac{140}{11} \simeq 13 \end{cases}$$

پس زاویه اولین بار $\frac{140}{11}$ دقیقه بعد از ۳ برابر ۲۰ درجه می‌شود و بار دوم در $3:20'$ برابر 20°



۵ - با توجه به شکل داریم:

$-\frac{1}{2} < \sin\theta \leq 1$

$\Rightarrow -\frac{1}{2} < \frac{m-3}{5} \leq 1 \Rightarrow +0/5 < m \leq 8$

$\Rightarrow$ اعداد صحیح $= \{1,2,3,4,5,6,7,8\} \Rightarrow 8$ عدد صحیح

۶ -

$$\frac{\sin(105°)+k\cos(255°)}{\cos(195°)+2\sin(195°)} = \frac{\sin(90+15)+k\cos(270-15)}{\cos(180-15)+2\sin(180+15)}$$

$$= \frac{\cos 15° - k\sin 15}{-\cos 15° - 2\sin 15} \xrightarrow{\div \cos 15} \frac{1-k\tan 15}{-1-2\tan 15} = \frac{1-\frac{k}{4}}{-1-\frac{1}{2}} = \frac{1-\frac{k}{4}}{-\frac{3}{2}} = 3$$

$$\Rightarrow 1-\frac{k}{4} = -\frac{9}{2} \Rightarrow \frac{k}{4} = \frac{11}{2} \Rightarrow k = 22$$

۷ -

$$\frac{1-\cos x}{1+\cos x} = 2 \Rightarrow 1-\cos x = 2+2\cos x \Rightarrow 3\cos x = -1 \Rightarrow \cos x = -\frac{1}{2}$$

زیرا x در ربع دوم است پس $\tan x < 0$

$$\Rightarrow \tan^2 x + 1 = \frac{1}{\cos^2 x} = \frac{1}{\frac{1}{9}} = 9 \Rightarrow \tan^2 x = 8 \Rightarrow \tan x = \pm 2\sqrt{2}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{-\sqrt{8}} = \frac{-\sqrt{8}}{8}$$

$$\frac{\tan(\frac{\pi}{2}-x) - \cot(\pi-x)}{\cot(\frac{3\pi}{2}-x) + 2\tan(2\pi-x)} = \frac{\cot x + \cot x}{\tan x - 2\tan x} = \frac{2\cot x}{-\tan x} = -2\cot^2 x$$

$$= -2\left(\frac{-\sqrt{8}}{8}\right)^2 = \frac{-2}{8} = -\frac{1}{4}$$

۸ -

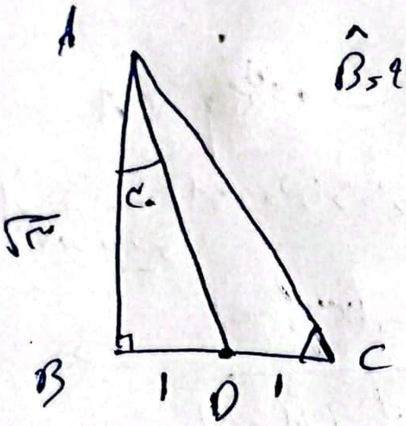
$$\frac{108°}{180} = \frac{\theta}{\pi} \Rightarrow \theta = \frac{108\pi}{180} = \frac{3\pi}{5}$$

$$\Rightarrow l = \theta R \Rightarrow l_A = l_B \Rightarrow \theta_A R_A = \theta_B R_B \Rightarrow \frac{3\pi}{5} R_A = \frac{9\pi}{10} R_B$$

$$\Rightarrow \frac{R_A}{R_B} = \frac{3}{2}$$

$$\Rightarrow \frac{S_A}{S_B} = \left(\frac{R_A}{R_B}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

۹-



$$\hat{B} = 90^\circ \Rightarrow AB = BD \cot 30 = BD \times \sqrt{3} = \sqrt{3}$$

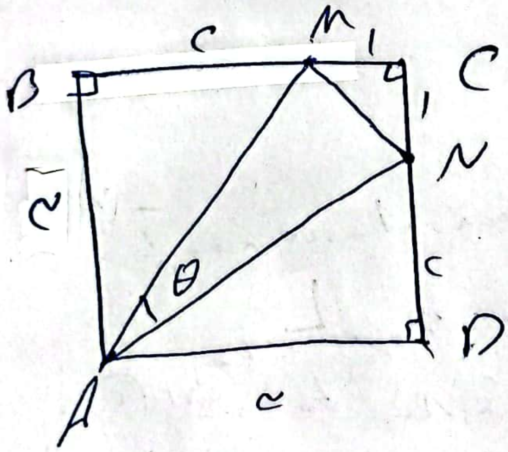
$$S = \frac{AB \times BC}{2} = \frac{\sqrt{3} \times BC}{2} = \sqrt{3} \Rightarrow \boxed{BC = 2}$$

$$\Rightarrow CD = 1$$

$$\Rightarrow \tan \hat{C} = \frac{AB}{BC} = \frac{\sqrt{3}}{2}$$

$$\sin 2\hat{C} = \frac{2\tan \hat{C}}{1 + \tan^2 \hat{C}} = \frac{\sqrt{3}}{1 + \frac{3}{4}} = \frac{\sqrt{3}}{\frac{7}{4}} = \frac{4\sqrt{3}}{7}$$

۱۰-



$$S_{\square ABCD} = 3 \times 3 = 9$$

$$S_{\triangle MCN} = \frac{1 \times 1}{2} = \frac{1}{2}$$

$$S_{\triangle AND} = \frac{3 \times 2}{2} = 3$$

$$S_{\triangle MBA} = \frac{3 \times 2}{2} = 3$$

$$\Rightarrow S_{\triangle AMN} = S_{ABCD} - S_{MCN} - S_{AND} - S_{MBA} = 9 - 3 - 3 - \frac{1}{2} = \frac{5}{2}$$

$$\Rightarrow S_{\triangle AMN} = \frac{1}{2} AN \times AM \times \sin\theta = \frac{1}{2} \times \sqrt{13} \times \sqrt{13} \times \sin\theta = \frac{5}{2}$$

$$\Rightarrow \frac{13}{2} \sin\theta = \frac{5}{2} \Rightarrow \sin\theta = \frac{5}{13}$$

$$AN = AM = \sqrt{2^2 + 3^2} = \sqrt{13}$$: AN, AM طول ★