به نام خدا

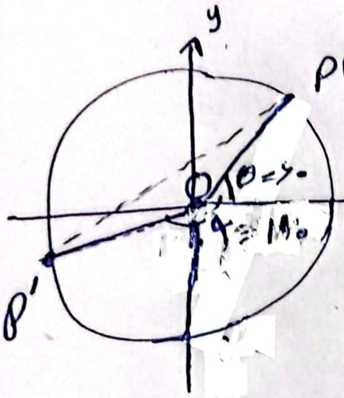
ماهان ترازکرد — دوازدهم ریاضی B — تکلیف ۱۲

۱- $\dfrac{a\sin\frac{4\pi}{3}+b\cos\frac{7\pi}{6}}{a\sin\frac{11\pi}{6}+b\cos\frac{\pi}{3}} = \dfrac{-\frac{\sqrt3}{2}a-\frac{\sqrt3}{2}b}{-\frac12 a+\frac12 b} = \dfrac{-\frac{\sqrt3}{2}(a+b)}{-\frac12(a-b)} = -\dfrac{\sqrt3}{2}$

$\Rightarrow -\sqrt3\dfrac{(a+b)}{a-b}=\dfrac{\sqrt3}{2} \Rightarrow \dfrac{a+b}{a-b}=\dfrac{-1}{2} \Rightarrow 2a+2b=b-a$

$\Rightarrow 3b=-3a$

$\Rightarrow \dfrac{b}{a}=\dfrac{-3}{1}=-3$



۲- $P(\frac12, y) \Rightarrow x_P = r\cos\theta = 1\cos\theta$

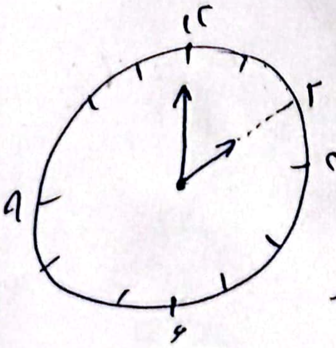
$\Rightarrow \cos\theta=\frac12 \Big\{ \Rightarrow \theta=60^\circ \xrightarrow{\text{دوران } 210^\circ} \theta'=-150^\circ$

θ در ربع اول

$\hat{POP'} = 360-60-150=150^\circ$

$\Rightarrow PP'=\sqrt{OP^2+OP'^2-2\,OP\cdot OP'\cos150^\circ}=\sqrt{2+\sqrt3}$

$\Rightarrow$ محیط $= P = OP+OP'+PP' = 2+\sqrt{2+\sqrt3}$



۳- از آنجایی که کل ساعت ۳۶۰ درجه و ۲π رادیان است

پس داریم: $\dfrac{2\pi}{12\times60'}=\dfrac{\frac{\pi}{9}}{x} \Rightarrow x=40'$

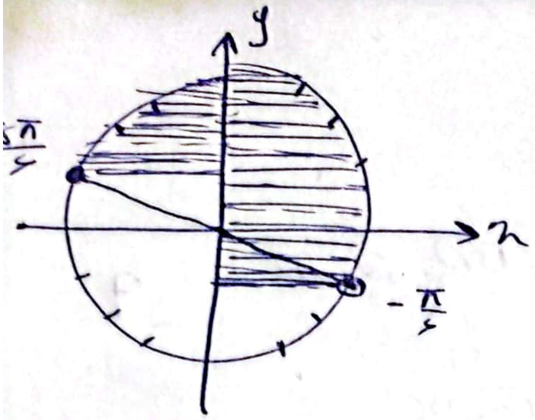
پس با $\frac{\pi}{9}$ حرکت عقربه ساعت شمار ۴۰' طی میشود

پس ساعت ۲:۴۰' است.

۴. $|5.5M-30H|=20 \Rightarrow |5.5M-90|=20$

$\Rightarrow \begin{cases} 5.5M-90=20 \Rightarrow 5.5M=110 \Rightarrow \boxed{M=20} \Rightarrow 3:20 \\ 5.5M-90=-20 \Rightarrow 5.5M=70 \Rightarrow M=\frac{140}{11}\approx 13 \end{cases}$

پس زاویه اولش ۳:۲۰' برابر ۲۰ درجه میشود و بار دوم ۱۴۰/۱۱ دقیقه بعد ۳ برابر ۲۰



د - با توجه به شکل داریم:

$-\frac{1}{2} < \sin\theta \le 1$

$\Rightarrow -\frac{1}{2} < \frac{m-3}{5} \le 1 \Rightarrow +0.5 < m \le 8$

$\Rightarrow$ اعداد صحیح $= \{1, 2, 3, 4, 5, 6, 7, 8\} \Rightarrow 8$ عدد صحیح

ه -

$$\frac{\sin(105^\circ) + k\cos(255^\circ)}{\cos(195^\circ) + 2\sin(195^\circ)} = \frac{\sin(90+15) + k\cos(270-15)}{\cos(180-15) + 2\sin(180+15)}$$

$$= \frac{\cos 15^\circ - k\sin 15}{-\cos 15^\circ - 2\sin 15} \xrightarrow{\div \cos 15} \frac{1 - k\tan 15}{-1 - 2\tan 15} = \frac{1 - \frac{k}{4}}{-1 - \frac{1}{2}} = \frac{1 - \frac{k}{4}}{-\frac{3}{2}} = 3$$

$$\Rightarrow 1 - \frac{k}{4} = -\frac{9}{2} \Rightarrow \frac{k}{4} = \frac{11}{2} \Rightarrow k = 22$$

و -

$$\frac{1 - \cos x}{1 + \cos x} = 2 \Rightarrow 1 - \cos x = 2 + 2\cos x \Rightarrow 3\cos x = -1 \Rightarrow \cos x = -\frac{1}{3}$$

نتیجه: x در ربع دوم است پس $\tan x < 0$

$$\Rightarrow \tan^2 x + 1 = \frac{1}{\cos^2 x} = \frac{1}{\frac{1}{9}} = 9 \Rightarrow \tan^2 x = 8 \Rightarrow \tan x = -2\sqrt{2}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{-\sqrt{8}} = \frac{-\sqrt{8}}{8}$$

$$\frac{\tan(\frac{\pi}{2} - x) - \cot(\pi - x)}{\cot(\frac{3\pi}{2} - x) + 2\tan(2\pi - x)} = \frac{\cot x + \cot x}{\tan x - 2\tan x} = \frac{2\cot x}{-\tan x} = -2\cot^2 x$$

$$= -2\left(\frac{-\sqrt{8}}{8}\right)^2 = \frac{-2}{8} = -\frac{1}{4}$$

۸ -

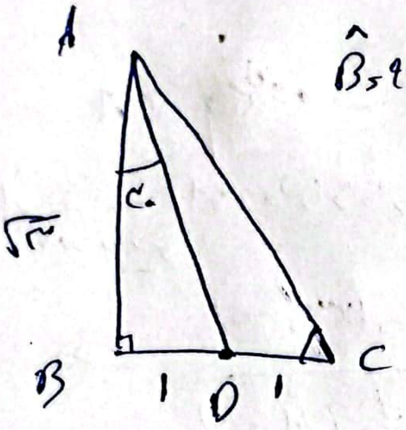
$$\frac{108^\circ}{180} = \frac{\theta}{\pi} \Rightarrow \theta = \frac{108\pi}{180} = \frac{3\pi}{5}$$

$$\Rightarrow L = \theta R \Rightarrow L_A = L_B \Rightarrow \theta_A R_A = \theta_B R_B \Rightarrow \frac{3\pi}{5} R_A = \frac{9\pi}{10} R_B$$

$$\Rightarrow \frac{R_A}{R_B} = \frac{3}{2}$$

$$\Rightarrow \frac{S_A}{S_B} = \left(\frac{R_A}{R_B}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

9-



$$\hat{B} = 90^\circ \Rightarrow AB = BD \cot 30^\circ = BD \times \sqrt{3} = \sqrt{3}$$

$$S = \frac{AB \times BC}{2} = \frac{\sqrt{3} \times BC}{2} = \sqrt{3} \Rightarrow \boxed{BC = 2}$$

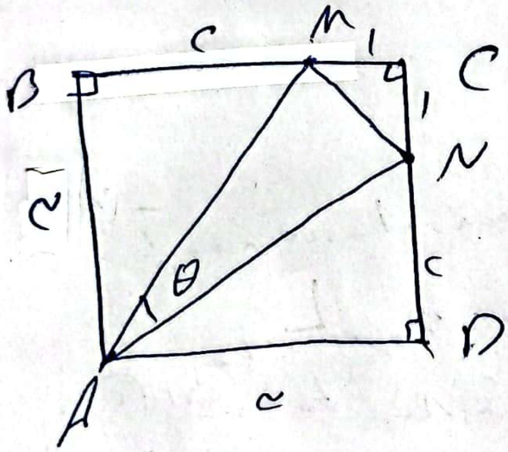
$$\Rightarrow CD = 1$$

(2)

$$\Rightarrow \tan \hat{C} = \frac{AB}{BC} = \frac{\sqrt{3}}{2}$$

$$\sin 2\hat{C} = \frac{2\tan\hat{C}}{1 + \tan^2\hat{C}} = \frac{\sqrt{3}}{1 + \frac{3}{4}} = \frac{\sqrt{3}}{\frac{7}{4}} = \frac{4\sqrt{3}}{7}$$

10-



$$S_{ABCD} = 3 \times 3 = 9$$

$$S_{MCN} = \frac{1 \times 1}{2} = \frac{1}{2}$$

$$S_{AND} = \frac{2 \times 3}{2} = 3$$

$$S_{MBA} = \frac{3 \times 2}{2} = 3$$

(2)

$$\Rightarrow S_{AMN} = S_{ABCD} - S_{MCN} - S_{AND} - S_{MBA} = 9 - 3 - 3 - \frac{1}{2} = 2.5$$

$$\Rightarrow S_{AMN} = \frac{1}{2} AN \times AM \times \sin\theta = \frac{1}{2} \times \sqrt{13} \times \sqrt{13} \times \sin\theta = 2.5$$

$$\Rightarrow \frac{13}{2} \sin\theta = \frac{5}{2} \Rightarrow \sin\theta = \frac{5}{13}$$

$$AN = AM = \sqrt{2^2 + 3^2} = \sqrt{13}$$: طول AN, AM ★