

$$y = \frac{2x+1}{3} \text{ و } y = \frac{2x-1}{3} \text{ داریم} \Rightarrow \frac{2x+1}{3} = \frac{2x-1}{3} \Rightarrow 2x+1 = 2x-1 \Rightarrow 1 = -1 \text{ که نادرست است}$$

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۱

الف) $y = \sqrt{x-2} + 1 \Rightarrow y-1 = \sqrt{x-2} \Rightarrow (y-1)^2 = x-2 \Rightarrow x = (y-1)^2 + 2 = y^2 - 2y + 3$
 $D_f = [1, \infty)$ $R_f = [2, \infty)$

ب) تابع معکوس $f^{-1}(y) = \sqrt{y-2} + 1$
 $D_{f^{-1}} = R_f = [2, \infty)$ $R_{f^{-1}} = D_f = [1, \infty)$

۲

$$f(x) = 2x - \sqrt{16 - 8x + x^2} = 2x - \sqrt{(x-4)^2} = 2x - |x-4|$$

$x > 4 \Rightarrow 2x - (x-4) = x+4 = f(x) \Rightarrow$ بار دینیز

$x < 4 \Rightarrow 2x - (4-x) = 3x-4 = f(x) \Rightarrow$ $x = \frac{y+4}{3} \Rightarrow y = \frac{3x-4}{3} = x - \frac{4}{3}$
 $D_{f^{-1}} = (-\infty, 4) \Rightarrow R_{f^{-1}} = (-\infty, 4)$

نمودار:

۳

۴

ضابطه: $a^2 - b^2 = (a-b)(a+b)$ باید آیه آیه باشد

$a^2 - 4a + 4 = (a-2)^2$ $a^2 - 4a + 4 = (a-2)^2$

۵

$$y = f(m + \frac{1}{\lambda} n) \Rightarrow f(m) + \frac{1}{\lambda} n$$

$$a > 0 \Rightarrow r m + a \leq f m \Rightarrow f^{-1}(a) \leq \frac{1}{r} m$$

$$D_f^{-1}(a) = \emptyset$$

$$m < 0 \Rightarrow r m + a \leq f m \Rightarrow f^{-1}(a) \leq \frac{1}{r} m$$

$$D_f^{-1} = (-\infty, 0]$$

$$y = \frac{f m}{\lambda} + \left| \frac{1}{\lambda} m \right|$$

$$\frac{r}{\lambda} \times \frac{1}{\lambda} = \boxed{\frac{r}{\lambda^2}}$$

6

$$f(r) = \frac{r+f}{r+m} = -1 \Rightarrow \boxed{m = -r} \quad y = \frac{r m + f}{m - r}$$

$$f \circ f^{-1}(a) = a$$

$$\Rightarrow y = m + f^{-1}(m) = a \Rightarrow f^{-1}(a) = 0 \Rightarrow a = f(0) = \frac{-f}{r}$$

7

$$q = r \quad g\left(\frac{r}{f}\right) = g(r) \Rightarrow r - f(r - f m) = g(m)$$

$$t =$$

8

$$f(m) = \frac{a(m-r) - r}{1-(a-r)} = r = \frac{am - r^2 - a + r^2}{r-a} = \frac{am - r^2}{r-a} = (r, 0), \text{ since } f^{-1}, f \text{ are bijections}$$

$$\frac{am - r^2}{r-a} = a \quad am - r^2 = r(a-r) \Rightarrow a^2 + am - r^2 = 0$$

$$a = \frac{-a \pm \sqrt{r^2 + 4m}}{2} \quad \boxed{\frac{-a \pm \sqrt{4ar}}{2}}$$

9

$$f^{-1}(a) = q \quad f(m) = a + f[m] \Rightarrow [m] + f[m] = [y] \Rightarrow [y] = [m] + f[m] = a[m] \Rightarrow \frac{1}{a}[y] = [m] \Rightarrow [m] = y - a$$

$$\frac{1}{a}[y] = y - a \Rightarrow a = \left[y - \frac{1}{a} y \right] \Rightarrow a = \frac{1}{a} [y] \Rightarrow y = g(a)$$

$$(f \circ f)(r) = a + f[m] = a + \frac{1}{a}[m] = \boxed{\frac{r}{a}[m]}$$

10