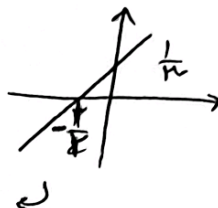


$$\frac{2}{3}x - \frac{1}{2} = f \rightarrow f^{-1} \rightarrow \frac{2}{3}(x + \frac{1}{2}) = \frac{2}{3}x + \frac{1}{3} \rightarrow$$

$$\frac{1}{2} \times \frac{2}{3} \times \frac{1}{2} = \boxed{\frac{4}{12}}$$



الف) $(x^2 + 4x + 3) \rightarrow$ $\rightarrow$ تابع یادمند یک به یک است $\rightarrow f^{-1}(x) = \boxed{1 + \sqrt{x-2}}$

ب) $x^2 - 2x + 3 \rightarrow$ $\rightarrow$ یک به یک نیست $\rightarrow f^{-1}(x) = \boxed{1 + \sqrt{x-2}}$ و $x \in [2, +\infty)$

$f(x) = 2x - 2|x-2| \rightarrow$ $\rightarrow x \in (-\infty, 2] \rightarrow$ واحد نیست

$\hookrightarrow x^2 - 2x + 2x - 2 = 2x - 2 \rightarrow$ $\hookrightarrow f^{-1}(x) = \frac{1}{2}x + 1 \rightarrow \frac{1 \times 2}{2} = \boxed{1}$

$f(x_1) = y_1$ و $f(x_2) = y_2$ و $f(x_1) = f(x_2)$ $\rightarrow D_f \in (-\infty, 0) \cup (0, +\infty)$

$\hookrightarrow \frac{x_1}{\sqrt{x_1^2 - 5}} = \frac{x_2}{\sqrt{x_2^2 - 5}} \rightarrow x_1^2 = x_2^2 \rightarrow x_1 = \pm x_2 \rightarrow$ تابع یک به یک نیست

$$f^{-1}(x) = -x \sqrt{\frac{5}{x^2 - 1}}$$

$g(2) = k - 2$

$g(x) \rightarrow x < 2 \rightarrow -x^2 + 4x + k \rightarrow 1 + 3k$

$\begin{cases} 1 + 3k < k - 2 \\ 2k - 12 \\ \boxed{k < -2} \end{cases}$

$$\frac{r_0}{r_0 + \frac{1}{2}x} \rightarrow f^{-1} \rightarrow \frac{\frac{1}{2}x}{\frac{1}{2}x + \frac{1}{2}x} \rightarrow a = \frac{r}{\lambda} \Rightarrow b = -\frac{1}{\lambda}$$

$$\frac{0}{(a-b)x + (a+b)x} \rightarrow \frac{r}{\lambda}x - \frac{1}{\lambda} \mid x$$

$$\rightarrow \boxed{ab = -\frac{r}{4\lambda}}$$

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$$f^{-1}(x) + f \circ f^{-1}(x) = \frac{rx + \varepsilon}{x - r} \quad x = x \rightarrow \boxed{x = -\frac{r}{r}}$$

$$\frac{y}{r+m} = \frac{-1}{x} \rightarrow r-m = xr \rightarrow m = -r$$

$$\rightarrow \frac{rx + \varepsilon}{x - r} = y \rightarrow f^{-1}(x) = \frac{rx + \varepsilon}{x - r}$$

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$$f(r-rx) = r - g\left(\frac{x}{r}\right) \rightarrow \underbrace{g\left(\frac{x}{r}\right)}_a + \underbrace{f(r-rx)}_b = r \rightarrow a+b=r$$

$$r g^{-1}(a) + f^{-1}(b) = \textcircled{r} \rightarrow \begin{matrix} a+b=r \\ \downarrow \quad \downarrow \\ r+(-r)=r \end{matrix}$$

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$$-f = \frac{-ax - 1r}{1+x} \rightarrow f = \frac{ax + 1r}{x+1} \xrightarrow{(x-r)} \frac{ax + 1r}{x-1} \rightarrow -r \rightarrow \frac{rx + y}{x-1} = f(x)$$

$$f^{-1}(x) = \frac{x+y}{x-r} \rightarrow f^{-1}(x) = f(x) \rightarrow \frac{x+y}{x-r} = \frac{rx+y}{x-1} \rightarrow \underbrace{x^2 + ax - y = rx^2 + rx - y}_{x^2 - rx - y}$$

$$S = -\frac{b}{a} = \textcircled{r}$$

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$$g(x) = f^{-1}(x) \rightarrow x + \varepsilon[x] \rightarrow \text{نفسه} \rightarrow \begin{matrix} r \rightarrow b \\ r \rightarrow a \\ r \rightarrow r \end{matrix} \left. \vphantom{\begin{matrix} r \rightarrow b \\ r \rightarrow a \\ r \rightarrow r \end{matrix}} \right\} x + \frac{r}{\omega} x = f^{-1}(x)$$

$$(f-g)(x) = x + r[x] - x + \frac{r}{\omega}[x] = \boxed{\frac{r}{\omega}[x]}$$

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