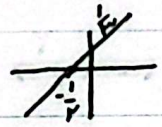
(۱
$\to 2y = 3x - 1 \to 3x - 2y = 1 \xrightarrow[\text{وارون}]{} 3y - 2x = 1$

$x = 0 \to y = \frac{1}{3}$
$y = 0 \to x = -\frac{1}{2}$

$\to S = \frac{\frac{1}{2} \times \frac{1}{3}}{2} = \frac{1}{12}$

(۲
الف) $y = (x-1)^2 + 2 ; x \geq 1 \to (x-1)^2 = y - 2 \to \sqrt{y-2} = |x-1|$

$x \geq 1 \to x - 1 = \sqrt{y-2} \to x = 1 + \sqrt{y-2} \xrightarrow[y \geq 2 \text{ و } x \geq 1]{\text{وارون}} y = 1 + \sqrt{x-2} = f^{-1}(x)$
$D_{f^{-1}} = [2, +\infty)$

$y_1 = y_2 \to (x_1-1)^2 + 2 = (x_2-1)^2 + 2 \to |x_1 - 1| = |x_2 - 1| \to x_1 = x_2$
$x_1, x_2 \geq 1$

ب) $y_1 = y_2 \to (x_1-1)^2 + 2 = (x_2-1)^2 + 2 \to |x_1 - 1| = |x_2 - 1| \to x_1 = x_2$
$x_1, x_2 \geq 2 > 1$

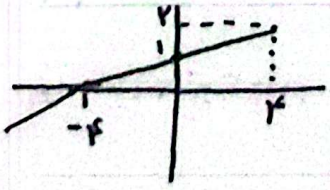
$x \geq 2 \to (x-1) \geq 1 \to x^2 - 2x + 1 \geq 4 \to x^2 - 2x + 3 \geq 6 \to y \geq 6$

$y = (x-1)^2 + 2 \to |x-1| = \sqrt{y-2} \to x = 1 + \sqrt{y-2} \to y = 1 + \sqrt{x-2}$
$D_{f^{-1}} = [6, +\infty)$

(۳
$y = 2x - \sqrt{16 - 16x + 4x^2} = 2x - \sqrt{4(x-2)^2} = 2x - 2|x-2|$

$y = \begin{cases} 4 ; x \geq 2 \\ 4x - 4 ; x \leq 2 \end{cases}$ تابع ثابت وارون ندارد $\to y = 4x - 4 ; x \leq 2 \to y \leq 4$

$\hookrightarrow x = \frac{y+4}{4} \to y = \frac{x+4}{4}$ وارون $D_{f^{-1}} = (-\infty, 4]$



$S = \frac{3+4}{2} = 6$ دو نقطه

(۴
$y = \frac{x}{\sqrt{x^2 - 5}} \to y_1 = y_2 \Rightarrow \frac{x_1}{\sqrt{x_1^2 - 5}} = \frac{x_2}{\sqrt{x_2^2 - 5}} \to \frac{x_1^2}{x_1^2 - 5} = \frac{x_2^2}{x_2^2 - 5}$
$x_1 \longleftrightarrow x_2$ هم علامت

$\to x_1^2 x_2^2 - 5x_1^2 = x_1^2 x_2^2 - 5x_2^2 \to |x_1| = |x_2| \xrightarrow[\text{هم علامت}]{} x_1 = x_2$

$y^2 = \frac{x^2}{x^2 - 5} \to x^2 = \frac{5y^2}{y^2 - 1} \to |x| = \frac{|y|\sqrt{5}}{\sqrt{y^2 - 1}} \xrightarrow[\text{هم علامت}]{} x = \frac{y\sqrt{5}}{\sqrt{y^2 - 1}}$

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۵) $x \geq 2 \rightarrow x-2 \geq 0 \rightarrow (x-2)^2 \geq 0 \rightarrow x^2-4x+4 \geq 0$

$\rightarrow x^2-4x+k \geq k-4$ ①

$x<2 \rightarrow (x-2)<-1 \rightarrow (x-2)^2>1 \rightarrow x^2-4x+4>1$

$\rightarrow -x^2+4x<3 \rightarrow -x^2+4x+3k<3+3k$ ②

$k-4 \geq 3k+3 \rightarrow 2k \leq -12 \rightarrow k \leq -6 \rightarrow k \in (-\infty, -6]$

① حداقل برای $x \geq 2$ ② حداکثر $x<2$

۶) $f(x)=2x+|x|=\begin{cases} 3x & x \geq 0 \\ x & x \leq 0 \end{cases} \rightarrow f^{-1}(x)=\begin{cases} \frac{x}{3} & x \geq 0 \\ x & x \leq 0 \end{cases}$

$g(x)=ax+b|x| \begin{cases} (a+b)x & x \geq 0 \\ (a-b)x & x \leq 0 \end{cases} \rightarrow f^{-1}(x)=g(x) \rightarrow \begin{matrix} a+b=\frac{1}{3} \\ a-b=\frac{1}{2} \end{matrix}$

$\rightarrow a=\frac{5}{12}, b=-\frac{1}{12} \rightarrow ab=\frac{-5}{144}$

۷) $f(x)=\frac{3x+4}{x+m} \rightarrow f(2)=\frac{10}{m+2}=-10 \rightarrow m=-3$

$f(x)=\frac{3x+4}{x-3} \longrightarrow y=\underbrace{f \circ f^{-1}(x)}_{x}+f^{-1}(x), \; y=x$

$\rightarrow x=x+f^{-1}(x) \rightarrow f^{-1}(x)=0 \rightarrow x=f(0)=-\frac{4}{3}$

۸) $t=g(\frac{x}{2}) \Rightarrow x=2g^{-1}(t) \rightarrow f(3-2x)=2-t$

$\rightarrow 3-2x=f^{-1}(2-t)=3-4g^{-1}(t) \rightarrow f^{-1}(2-t)+4g^{-1}(t)=3$

$t=-2 \rightarrow f^{-1}(4)+4g^{-1}(-2)=3$

۹) $y=\frac{5x-13}{1-x} \xrightarrow[\text{قرینه حول } y=x]{} y=\frac{5x+13}{x+1} \xrightarrow[\text{۲ به راست}]{} y=\frac{5x+3}{x-1} \xrightarrow[\text{۲ به بالا}]{} y=\frac{5x+5}{x-1}$

$f(x)=\frac{2x+5}{x-1} \longrightarrow f^{-1}(x)=\frac{x+5}{x-2} \rightarrow f^{-1}(x)=f(x) \rightarrow \frac{5x+5}{x-1}=\frac{x+5}{x-2}$

$\frac{2x+5}{x-1}=\frac{x+5}{x-2} \longrightarrow \frac{x}{1}=\frac{x+5}{x-2} \rightarrow x^2-2x=x+5 \rightarrow x^2-3x-5=0$

↳ جمع مقادیر $=3$

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(1.

$f(x) = x + 4[x] = y \rightarrow [y] = [x] + 4[x] = 5[x]$

$y = x + 4[x] = x + \frac{4}{5}[y] \rightarrow x = y - \frac{4}{5}[y]$

$\rightarrow$ وارون $y = x - \frac{4}{5}[x] = g(x) \rightarrow f - g = (x + 4[x]) - (x - \frac{4}{5}[x])$

$= \frac{24}{5}[x]$